English version

Introduction to Symplectic Geometry and Deformation Quantization

Level of course
PhD Course

Semester/quarter
3rd + 4th quarter (Spring 2010)

Hours per week
4

Name of lecturer
Hans-Christian Herbig

Objectives of the course
This is an announcement of an introductory course on symplectic manifolds from the point of view of quantization theory. A highlight will be the celebrated result of Fedosov which gives a (in a sense universal) construction of star products on symplectic manifolds.

Prerequisites
As a prerequisite we assume acquaintance with basic notions from differential geometry (such as vector bundles, de Rham cohomology, connections) and Lie groups (left invariant vector fields, exponential map, closed subgroups). Some experience with homological algebra is desirable but not necessary.

Course contents
The course will encompass the following topics:

1. Basic notions of symplectic geometry:

   - symplectic forms, symplectomorphism, symplectic vector fields, symplectic volume, Poisson bracket, Hamitonian vector fields, Hamiltonian equations of motion,
   - the Darboux theorem,

- more special situations: cotangent bundles, Kähler manifolds, coadjoint orbits,
- basic notions from Lie groups actions: fundamental vector fields, free actions, proper actions, isotropy groups etc.
- moment maps, Hamiltonian reduction.

2. The problem of quantization:

    - observables, states etc. in the realm of classical and quantum mechanics,
    - the Groenewald-van-Howe theorem,
    - quantization as a deformation:
        - deformations of associative algebras in the sense of Gerstenhaber,
        - algebraic structures on the Hochschild co-chain complex of an associative algebra, Maurer-Cartan equation of a differential graded Lie algebra,
        - the Gerstenhaber algebra of polyvector fields, Poisson tensors, Poisson cohomology,
        - the Hochschild-Kostant-Rosenberg theorem.

3. Fedosov's construction of star products on a symplectic manifold:

    - Koszul resolution, symplectic connections, a perturbation lemma,
    - recursive construction of the Fedosov differential,
    - Fedosov's Taylor series and Fedosov's star product,
    - a version of Cartan's magic formula and N. Neumaier's classification of quantum moment maps.

Literature

A big part of the material is covered by the book [1], which is unfortunately written in German. An important source on deformation quantization of symplectic manifolds is the book of Fedosov [2]. There are meanwhile many textbooks on symplectic geometry and geometric mechanics – let us mention [3], [4], [5] and of course [6].

References

[1] S. Waldmann, *Poisson-Geometrie und Deformationsquantisierung. Eine Einführung.*, Springer-Verlag, Berlin, xii+612 pp.

[2] B. Fedosov, *Deformation quantization and index theory*, Mathematical Topics, 9. Akademie Verlag, Berlin, 1996. 325 pp.

[3] R. Abraham and J. E. Marsden *Foundations of mechanics*, Second edition, revised and enlarged. With the assistance of Tudor Ratiu and Richard Cushman. Benjamin/Cummings Publishing Co., Inc., Advanced Book Program, Reading, Mass., 1978. xxii+m-xvi+806 pp.

[4] J. E. Marsden and T. S. Ratiu, *Introduction to mechanics and symmetry. A basic exposition of classical mechanical systems.*, Second edition, Texts in Applied Mathematics, 17. Springer-Verlag, New York, 1999. xviii+582 pp.

[5] A. Cannas da Silva, *Lectures on symplectic geometry*, Lecture Notes in Mathematics, 1764. Springer-Verlag, Berlin, 2001. xii+217 pp.

[6] V.I. Arnold, *Mathematical methods of classical mechanics.* Translated by K. Vogtman and A. Weinstein, Graduate Texts in Mathematic. 60, Springer-Verlag, 462 pp.

Teaching methods
4 hours of lectures per week.

Assessment methods
Passed / not passed will be based on the students participation in the course.

Credits
10 ECTS

Language of instruction
English

Course enrolment
Please send an e-mail to Maiken Nielsen, maiken@imf.au.dk