Spring 2019 - Q3+4

PhD course:

Quivers and moduli problems in algebraic geometry

Brief description

A quiver Q is a directed graph where loops and multiple arrows between edges are allowed. A representation of Q is an assignment of a vector space to each vertex of Q and a linear mapping between the corresponding vector spaces to each arrow. While this seems like a simple combinatorial tool for organizing linear algebraic data, it is related to many interesting objects in both representation theory and algebraic geometry. In particular, quiver representations have been studied from the geometric point of view in terms of moduli spaces and moduli stacks. The aim of this course is to use several key examples in order to provide an introduction to quiver-related “solutions" for moduli problems in algebraic geometry.

We begin with a brief overview of quiver representations and moduli functors, discussing fine and course moduli spaces, as well as stacks. We continue with a description of framed vector bundles on the projective line in terms of representations of the Kronecker quiver and framed torsion-free sheaves on the projective plane in terms of the ADHM quiver. Using the former, we construct a moduli space of quasi-parabolic bundles over the projective line. We will mention how this moduli space relates to the corresponding moduli stack.

We introduce quiver varieties by discussing symplectic geometry in the context of quiver representations and providing an explicit formula for the moment map associated with the standard group action on the cotangent bundle to the variety of quiver representations. We go on to define parabolic connections, and describe a moduli space of such connections over the projective line in terms of fibers of moment map coming from the representations of a quiver with relations.

We conclude by giving an overview of a quiver construction for irregular singular connections on trivial vector bundles over the projective line.

Requirement:

A familiarity with elementary algebraic geometry (the basics of varieties and sheaf cohomology) and category theory is extremely useful, but I will go over the required results where necessary. Later on, some introductory symplectic geometry (moment maps on cotangent bundles) would be helpful as well.

Lecturer: Alexander Soibelman & Jørgen Ellegaard Andersen

Duration: 2 x 2 lectures per week in Q3+4

Registration: ggm@au.dk

Schedule: Monday 12-14 & Thursday 15-17 in Koll. G (1532-214)