Moduli spaces of vector bundles on Riemann surfaces

Level of course
PhD Course

Semester/quarter
1st + 2nd quarter (Autumn 2010)

Hours per week
4

Name of lecturer
Johan Martens and Jørgen Ellegaard Andersen

Objectives of the course
The goal of this course is to give an introduction to the moduli space of bundles over a Riemann surface or algebraic curve.

Course contents
The goal of this course is to give an introduction to the moduli space of bundles over a Riemann surface or algebraic curve. These moduli spaces have been studied intensely in the last fifty years, and their rich structure can be investigated from various viewpoints - topological, differential geometric, algebraic geometric. In this course we shall mainly follow a gauge-theoretic approach, indicating the links with other paths along the way.

In particular we shall discuss the following:

- Review of bundles and Riemann surfaces, notion of stability.

- Complex analytic construction of the moduli-space of semi-stable bundles. Link with Yang-Mills.

- Morse-theoretic approach to Betti-numbers of the moduli space.

- Intersection theory.

- Stable bundles and representations of the fundamental group. Canonical symplectic structure on the moduli space.

Literature
We shall mainly follow original papers, discussing the necessary background as we go along. In particular the following references will be used:

Atiyah, M. F. and Bott, R., The Yang-Mills equations over Riemann surfaces. Philos. Trans. Roy. Soc. London Ser. A 308 (1983), no. 1505, 523–615.

Narasimhan, M. S. and Seshadri, C. S., Stable and unitary vector bundles on a compact Riemann surface. Ann. of Math. (2) 82 1965 540–567.

Thaddeus, Michael, An introduction to the topology of the moduli space of stable bundles on a Riemann surface. Geometry and physics (Aarhus, 1995), 71–99, Lecture Notes in Pure and Appl. Math., 184, Dekker, New York, 1997.

Goldman, William M., The symplectic nature of fundamental groups of surfaces. Adv. in Math. 54 (1984), no. 2, 200–225.

Le Potier, J., Lectures on vector bundles. Cambridge Studies in Advanced Mathematics, 54. Cambridge University Press, Cambridge, 1997.

Teaching methods
4 hours of lectures per week

Assessment methods
Passed / not passed will be based on the students participation in the course

Credits
10 ECTS

Language of instruction
English

Course enrolment
Please send an e-mail to Maiken Nielsen, maiken@imf.au.dk