Fall 2017

PhD course:

Hilbert schemes of points on surfaces

Brief description

The course is planned as a gentle introduction to ideas and methods in the study of moduli spaces in general, with tools coming from algebraic geometry and symplectic geometry. It is based on the book by Hiraku Nakajima "Lectures on Hilbert Schemes of Points on Surfaces". However, rather than just following the book, we will slowly introduce the objects of study from the point of view of moduli spaces. Thus no deep knowledge of algebraic geometry or differential geometry is expected from the audience.

We begin by talking about projective schemes in general, Grassmannians and Flag varieties. We pass to a concrete description of the Hilbert scheme of points on the complex plane in terms of linear algebra. The key words here are GIT, quivers and quiver varieties. Then we move to Hilbert schemes of points on a general surface and to the framed moduli space of torsion free sheaves on the projective plane. We define a symplectic structure on the defined moduli spaces.

Next we study Hilbert schemes of points on the plane from analytic point of view and introduce a canonical hyper-Kähler structure on it. We talk about the relation of GIT, symplectic reduction and hyper-Kähler quotients. In particular, we discuss how to obtain minimal resolutions of two dimensional Gorenstein quotient singularities using Hilbert schemes of points and their relations with Kronheimer's ALE hyper-Kähler metrics.

We then study in detail the topology of Hilbert schemes of points on surfaces and calculate the generating function for the cohomology.

Finally, we pass to Geometric Representation theory and construct a canonical action of the Heisenberg algebra on the direct sum of cohomology of Hilbert schemes of points on a surface.

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Lecturer: Sergey Arkhipov & Cristiano Spotti.

Duration: 2 x 2 lectures per week in the fall term 2017

Registration: qgm@au.dk

Schedule: Tuesdays 14-17 in Øv.G.33 (1532-322)

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