

PhD course

1st + 2nd quarter 2014 / 3rd + 4th quarter 2015

By Sergey Arkhipov and Henning Haahr-Andersen

Title:

Symplectic reflection algebras

Hours per week

2 standard lectures per week.

Objectives of the course

Symplectic reflection algebras were introduced by Etingof and Ginzburg about 10 years ago based on the previous work of Cherednik, Dunkl, Crawley-Boevey and Holland, and others. They happen to be connected to various parts of Mathematics: Representation theory (representations of quivers, classical and quantum Nakajima quiver varieties, Hecke algebras and categorical Kac-Moody actions), Algebraic geometry (resolutions of quotient singularities, geometry of plane curves) Combinatorics (Macdonald polynomials), Deformation theory, Integrable systems (systems of Calogero-Moser type), Knot theory (invariants of toric knots).

The main goal of this course is to demonstrate some of these connections based on elementary material and to make them accessible at a graduate student level.

Prerequisites

Elementary algebra and rudiments of algebraic geometry (including affine and projective algebraic varieties over complex numbers).

Course contents

1. Kleinian singularities and their deformations: finite subgroups of $SL_2(\mathbb{C})$, Algebras of Crawley-Boevey and Holland, Representations of quivers and categorical quotients, McKay correspondence, deformed preprojective algebras.
2. Construction of SRA (deformation theory).
3. Calogero-Moser systems and Cherednik algebras (integrable systems): Classical Calogero-Moser system, Hamiltonian reduction and Calogero-Moser space, Quantization as deformation – Quantum CM system, Dunkl operators and first integrals for quantum CM system.
4. Categories \mathcal{O} for Cherednik algebras, KZ functor, Hecke algebras, Induction and restriction functors.

Literature

There is basically one textbook on the subject and several review texts, all available online.

The textbook is: P. Etingof. Lectures on Calogero-Moser systems.

<http://arxiv.org/abs/math/0606233>

There are also notes from MIT class, 2009: P. Etingof, X. Ma. Lecture notes on Cherednik algebras.

<http://arxiv.org/abs/1001.0432>

One of the review texts: I. Gordon. Symplectic reflection algebras. <http://arxiv.org/abs/0712.1568>

Teaching methods

4 hours of lectures per week, with a few non-graded assignments.

Credits

10 ECTS

Language of instruction

English