Aarhus Gauge Theory Workshop

8-12 August 2011

Programme

Monday 8 August:

09.30-10.30	**Stefan Bauer**:	*Differential equations and stable homotopy*
11.00-12.00	**Nikolai Saveliev**:	*An index theorem for end-periodic opererators*
13.30-14.30	**Kim Frøyshov**:	*Smooth four-manifolds and intersection forms with local coefficients*
14.45-15.45	**Christopher Herald**:	*An SU (3) Casson invariant of rational homology spheres*
16.15-17.15	**Brendan McLellan**:	*Non-Abelian Localization and U (1) Chern-Simons Theory*
18.00-	Standing buffet in Math lab	

Tuesday 9 August:

09.30-10.30	**Jacob Rasmussen**:	*Khovanov homology of torus knots*
11.00-12.00	**Scott Baldridge**:	*On the rotation class of knotted Legendrian tori in R^5*
13.30-14.30	**Paul Kirk**:	*Instantons, concordance, and Whitehead doubling*
14.45-15.45	**Matthew Hedden**:	*The Khovanov module and unlink detection*
16.15-17.15	**Timothy Nguyen**:	*Lagrangians from Seiberg-Witten Theory and Donaldson's TQFT*

Wednesday 10 August:

09.30-10.30	**Andras Stipsicz**:	*Surface singularities with rational homology disk smoothings*
11.00-12.00	**Inanc Baykur**:	*Round handles and smooth four-manifolds*
13.30-14.30	**Burak Ozbagci**:	*Milnor fillable contact structures are universally tight*
14.45-15.45	**Brendan Owens**:	*A concordance group of links*
16.15-17.15	**Gabor Etesi**:	*On the existence of a complex structure on the six-sphere*

Thursday 11 August:

09.30-10.30	**Ron Fintushel**:	*Surgery on nullhomologous tori*
11.00-12.00	**Hans Boden**:	*An SU (N) Casson-Lin invariant for links $L \subset S^3$ with more than one component*
13.30-14.30	**Maxim Kontsevich**:	*Symplectic topology of complex integrable systems.*
14.45-15.45	**Andriy Haydys**:	*Fukaya-Seidel category and gauge theory*
16.15-17.15	**Henrique Sá Earp**:	*Perspectives on G_2–instantons*
17.45-	Special Dinner in Math Lab	

Friday 12 August:

09.30-10.30	**Jonathan Weitsman**:	TBA
11.00-12.00	**Celso Doria**:	*Variational aspects about the Existence of Seiberg-Witten Monopoles on a Four Manifold*

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Titles and Abstracts

Scott Baldridge: *On the rotation class of knotted Legendrian tori in* $\mathbf{R}^5$

In this talk I will discuss how to use Lagrangian hypercube diagrams to easily compute the "rotation number" for a large class of knotted Legendrian tori in $\mathbf{R}^5$ with respect to the standard contact form. This result is the first step in a program to understand the contact homology of such embeddings and is joint work with Ben McCarty.

Stefan Bauer: *Differential equations and stable homotopy*

The talk considers a space parametrizing certain differential equations. The topology of the space in question turns out to be related to spaces well known from stable homotopy theory: The classifying spaces of ko-theory and stable homotopy theory.

Inanc Baykur: *Round handles and smooth four-manifolds*

In this talk, we are going to unfold the strong affiliation of round handles with smooth four-manifolds. Various fundamental topics that appear in the study of four-manifolds, such as handlebodies, cobordisms, logarithmic transforms along tori, exotic smooth structures, and broken Lefschetz fibrations, will come into play as we discuss the relevant interactions between them.

Hans Boden: *An $SU(N)$ Casson-Lin invariant for links $L \subset S^3$ with more than one component.*

In 1992, X.-S. Lin introduced a Casson-type invariant $h(K)$ for knots $K \subset S^3$ that counts conjugacy classes of irreducible $SU(2)$ representations of the knot group G_K with meridional trace equal to -2. Lin identified $h(K)$ with the signature of the knot, and his approach was generalized to give invariants for other trace conditions and for knots in homology 3–spheres independently by C. Herald in 1997 and by M. Heusener and J. Kroll in 1998. In [Pac. J. Math. 248 (2010), 139-154], E. Harper and N. Saveliev define a Casson-Lin type invariant $h(L)$ for 2–component links $L \subset S^3$ and show that $h(L)$ equals the linking number. This talk is a report on recent joint work with Eric Harper introducing analogous invariants of links $L \subset S^3$ with $n \geq 2$ components using the group $SU(N)$. The invariants are denoted $h_{N,a}(L)$, where $a = (a_1, \ldots, a_n)$ is a n–tuple of integer labels, one for each component of the link, and they are defined as a signed count of conjugacy classes of certain projectively flat $SU(N)$ invariants of the link group G_L. The talk will outline the compactness and irreducibility results needed to show that $h_{N,a}(L)$ is well-defined, a vanishing result for split links, and preliminary computations.

Celso Doria: *Variational aspects about the Existence of Seiberg-Witten Monopoles on a Four Manifold*

In abscence of an existence theorem for Seiberg-Witten Monopoles on a smooth four manifold a Variational set up is used to study the question. There exist smooth 4-manifolds admitting monopoles, whose existence is achieved by proving the Seiberg-Witten invariant is non-trivial. However, in all cases known by the author, the invariant's non-triviality depends on the fact that

the invariant of a symplectic 4-manifold is non-trivial. Thus, by exploring the variational set up, the Witten's existence theorem on a Kähler manifold is proved and some question concerning the lowest eigenvalue of the differential operator $L_A = \nabla_A + \frac{k_g}{4}$, defined on sections of the positive spinors bundles, are discussed.

Gabor Etesi: *On the existence of a complex structure on the six-sphere*

Existence of a complex structure on the six dimensional sphere is re-examined in the talk. The approach is based on re-interpreting a hypothetical complex structure as a classical vacuum solution of a non-linear Higgs scalar field theory (with gauge symmetry) formulated on S^6. This classical vacuum solution is then constructed by Fourier expansion from the obvious one of a similar theory on the exceptional compact Lie group G_2.

Ron Fintushel: *Surgery on nullhomologous tori*

Abstract: I will discuss joint work with Ron Stern which attempts to identify, in a connect sum of projective spaces with either orientation, a single nullhomologous torus upon which surgery changes the smooth structure. This will be described more or less explicitly for the case of $\mathbf{C}P^2 \# 3(-\mathbf{C}P^2)$.

Kim Frøyshov: *Smooth four-manifolds and intersection forms with local coefficients*

Let X be a closed, oriented, smooth 4–manifold. We consider singular cohomology of X with coefficients in a bundle L of infinite cyclic groups over X. The cup product gives rise to a unimodular quadratic form Q_L on $H^2(X; L)/$torsion. If L is trivial then this is just the usual intersection form Q of X. In the 1980's Donaldson proved, using instanton moduli spaces, that if Q is definite then it must be diagonal. In this talk I will extend this result to Q_L for non-trivial L, under some constraints. This yields new examples of non-smoothable topological 4–manifolds.

Andriy Haydys: *Fukaya-Seidel category and gauge theory*

I will outline a new construction of the Fukaya-Seidel category, which is associated to a symplectic manifold equipped with a compatible almost complex structure J and a J–holomorphic Morse function. Then this construction will be applied in an infinite dimensional case of the complex Chern-Simons functional. The corresponding construction, which is based upon a five-dimensional gauge theory, conjecturally associates a Fukaya-Seidel-type category to a smooth three-manifold.

Matthew Hedden: *The Khovanov module and unlink detection*

Kronheimer and Mrowka recently showed that Khovanov homology detects the unknot using connections with an instanton Floer homology invariant for knots. I'll discuss a module structure on Khovanov homology, and a theorem that connects this structure with the Heegaard Floer module of the branched double cover of a link. Using this connection, together with a result indicating that the Heegaard Floer module detects homologically essential spheres in 3–manifolds, we can show that the Khovanov module detects (n–component) unlinks. This is joint work with Yi Ni.

Christopher Herald: *An $SU(3)$ Casson invariant of rational homology spheres*

In this talk I will describe joint work in progress with Hans Boden, which develops an $SU(3)$ Casson invariant of rational homology spheres. This invariant extends the integer-valued invariant of integral homology spheres defined in earlier work by the authors together with Paul Kirk. After perturbing the flatness equation to obtain a flat moduli space consisting of finitely many points, we begin with an algebraic count of the irreducible points, and then add suitable correction terms for the reducible points and two types of non-central abelian points. The correction terms are defined in terms of spectral flow of the odd signature operator, and have the property that the algebraic count of irreducible points plus the correction terms is perturbation independent.

Paul Kirk: *Instantons, concordance, and Whitehead doubling*

We use moduli spaces of $SO(3)$ instantons and Chern-Simons invariants of flat connections to show that the whitehead doubles of certain infinite families of torus knots are linearly independent in the kernel of the homomorphism from the smooth knot concordance group to the topological concordance group. (Joint with Matthew Hedden)

Maxim Kontsevich: *Symplectic topology of complex integrable systems*

Brendan McLellan: *Non-Abelian Localization and $U(1)$ Chern-Simons Theory*

Our goal in this talk is to describe the method of non-abelian localization and how this method yields some new results in $U(1)$ Chern-Simons theory. Starting from a beautiful localization result of Duistermaat and Heckman, we review localization on a finite dimensional Hamiltonian G–space and recall how this result is generalized to path integrals in quantum field theory. In particular, we briefly recall Edward Witten's 1992 paper, "Two dimensional Gauge Theories Revisted", where non-abelian localization is introduced and applied to two dimensional quantum Yang-Mills theory, and Chris Beasley's and Edward Witten's 2005 paper, "Non-Abelian Localization for Chern-Simons Theory". We will then present our results (joint with Lisa Jeffrey), which follow by adapting the method of non-abelian localization to $U(1)$ Chern-Simons theory.

Timothy Nguyen: *Lagrangians from Seiberg-Witten Theory and Donaldson's TQFT*

We discuss how boundary values of the space of solutions to the Seiberg-Witten equations, both on compact 3–manifolds and on 3–manifolds with cylindrical ends, yield Lagrangian submanifolds within the corresponding boundary con figuration space. In the case of cylindrical ends, this construction provides a Lagrangian correspondence between the vortex moduli spaces at infinity. As an application, we discuss work in progress for supplying the analytic details of Donaldson's TQFT construction of the Seiberg-Witten invariants of a closed 3–manifold.

Brendan Owens: *A concordance group of links*

I will discuss a notion of sliceness for links based on Euler characteristic and show that it leads to an equivalence relation generalising knot concordance. The equivalence classes form a group L containing the knot concordance group as a direct summand with infinitely generated direct complement. I will also exhibit some homomorphisms from L. This is joint work with Andrew Donald.

Burak Ozbagci: *Milnor fillable contact structures are universally tight*

We show that the canonical contact structure on the link of a normal complex singularity is universally tight. As a corollary we show the existence of closed, oriented, atoroidal 3-manifolds with infinite fundamental groups which carry universally tight contact structures that are not deformations of taut (or Reebless) foliations. This is a joint work with Y. Lekili.

Jacob Rasmussen: *Khovanov homology of torus knots*

Khovanov homology and its generalizations have proven to be very interesting invariants of knots in S^3, but their geometrical meaning remains mysterious. I'll discuss some conjectures relating Khovanov homology of a very special class of knots (torus knots) with algebraic geometry (Hilbert schemes of singular plane curves) and representation theory.

Henrique Sá Earp: *Perspectives on G_2–instantons*

Solutions to the Hermitian Yang-Mills problem over A. Kovalev's asymtotically cylindrical Calabi-Yau 3–folds induce instantons over compact 7–manifolds with holonomy group G_2, obtained by a twisted gluing procedure. Moreover, algebraic-geometric monad constructions can be used to generate numerous concrete examples of such G_2–instantons. I will present a survey of that study, punctuated by some open questions ranging from naive to quite ambitious.

Nikolai Saveliev: *An index theorem for end-periodic operators*

We extend the Atiyah, Patodi, and Singer index theorem for first order differential operators from the context of manifolds with cylindrical ends to that of manifolds with general periodic ends. This theorem provides a natural complement to Taubes' Fredholm theory for general end-periodic operators. It expresses the index in terms of a new periodic eta-invariant that equals the Atiyah-Patodi-Singer eta-invariant in the cylindrical setting. As an application, we use these index-theoretic techniques to make a count of Seiberg-Witten monopoles on a homology $S^1 \times S^3$ into a topological invariant. This is a joint project with Tomasz Mrowka and Daniel Ruberman.

Andras Stipsicz: *Surface singularities with rational homology disk smoothings*

We indicate the classification of weighted homogeneous singularities with rational homology disk smoothings, and show a few examples of such singularities. We examine the similar question in the symplectic category, and (in certain particular) cases show uniqueness for the symplectic fillings (up to symplectic deformation).

Jonathan Weitsman: *TBA*