QUANTIZATION OF SINGULAR SPACES
TALKS

Daniel Sternheimer, Rikkyo University (Tokyo) and Universit de Bourgogne

Title: A singular view on quantization

Abstract: In the Copenhagen interpretation of quantum theories, quantization manifests itself as a map (a functor) between a category of 'classical' observables ('functions' on some classical space) and a category of objects ('operators') acting in some Hilbert space. Many warned over time that "it ain't necessarily so". Deformation quantization gives a framework to get out of the dilemma, quantization being understood as a deformation of the classical (commutative) composition law of observables. It coincides with usual quantization when such a functor into the Procrustean bed of Hilbert space can be defined and permits generalizations which should play an important role, in particular when dealing with singular spaces. But these generalizations are often formal and not restrictive enough. It is therefore desirable to develop effective formalisms able to 'focus' the target of deformation quantization. In this talk we shall first present an overview of the main points of deformation quantization, its conceptual basis in the role of deformations in physics and its relations with usual quantization. We end by indicating some avenues susceptible to focus the 'quantization' part of deformation quantization, in particular in view of dealing with singular spaces.

Johannes Huebschmann, Université des Sciences et Technologies de Lille

Title: The moduli space of semi-stable holomorphic vector bundles on a curve revisited as a stratified Kähler space: The singular structure

Abstract: A well-known construction of Seshadri establishes the moduli space of semi-stable holomorphic vector bundles on a curve as a normal projective variety. A crucial tool is Grothendieck's Quot-scheme. Under suitable circumstances this variety is non-singular and inherits a Kähler structure. An infinite-dimensional approach due to Atiyah and Bott establishes the moduli space by a version of infinite-dimensional Kähler reduction. By means of a suitable extended moduli space, as a stratified symplectic space, the moduli space arises by symplectic reduction in finite dimensions. We will explain a construction of the moduli space as a Kähler quotient in finite dimensions. In this construction, we use a space of the kind to which Seshadri had applied the quot-construction. Among the tools is Chen's theory of differentiable spaces or, equivalently, Souriau's theory of "diffeological spaces". The construction explains in particular the in general singular structure of the moduli space as a stratified Kähler space: In a special case, a Kummer surface arises as the singular locus of an exotic stratified Kähler structure on projective 3-space. Here the term "exotic" is intended to refer to the fact that that structure on projective 3-space is essentially different from the standard (non-singular) Kähler structure.

Pierre Schapira, Université Paris VI,

Title: Deformation quantization modules

Abstract: We shall give an overview of recent results on modules over deformation quantization algebroids on complex Poisson manifolds. We study in particular the composition of kernels (finiteness and duality), the functoriality of Hochschild classes and the link with the Riemann-Roch theorem. Finally, we have a glance to holonomic modules on symplectic manifolds.

Hessel Posthuma, University of Amsterdam

Title: The higher index theorem on orbifolds.

Abstract: I will give an overview of joint work with Markus Pflaum and X. Tang on the higher index theorem for elliptic operators on orbifolds and generalizations of this theorem to more general situations.

Tudor Ratiu, École polytechnique fédérale de Lausanne

Title: Singular reduction, quantum reduction, and coherent states quantization

Abstract: In this talk I will review the singular point reduction process for general symplectic manifolds and some of the known results for singular cotangent bundle reduction. Then I will briefly present quantum reduction in the context of coherent states quantization. This will require the introduction of infinite dimensional Poisson geometry that will also be briefly reviewed.

Alexander Karabegov, Abilene Christian University

Title: Star products with separation of variables admitting a smooth extension

Abstract: Given a complex manifold M with an open dense subset Ω endowed witha pseudo-Kähler form ω which cannot be smoothly extended to a larger open subset, we consider various examples where the corresponding Kähler-Poisson structure and a star product with separation of variables on (Ω, ω) admit smooth extensions to M. We suggest a simple method to prove the existence of a smooth extension of a star product along with the corresponding Poisson structure and apply it to these examples. We expect that this method can be generalized to singular Kähler manifolds.

Xiang Tang, Washington University in St. Louis

Title: Mackey Machine and Duality of Gerbes on Orbifolds

Abstract: Let G be a finite group and Y a G-gerbe over an orbifold B. We will explain a construction of a new orbifold $\widehat{Y}$ and a flat $U(1)$-gerbe c on $\widehat{Y}$. Motivated by a proposal in physics, we study a mathematical duality of G-gerbes, which asserts that the geometry of Y is equivalent to the geometry of $\widehat{Y}$ twisted by c. The Mackey machine provides us the right tool to study such a problem. We will discuss some results in symplectic topology with the help of noncommutative geometry. This is a joint work with Hsian-hua Tseng.

Giuseppe Dito, Université de Bourgogne

Title: Star-products as a pseudo BCH-formula

Abstract: On $\mathbb{R}^n$ endowed with a Poisson tensor $\pi \in \Gamma(\wedge^2 T\mathbb{R}^n)$, there exists a deformation quantization algebra $K_\hbar$ defined by Kontsevich. When π is a polynomial Poisson structure, B. Shoikhet conjectured that the algebra $K_\hbar$ is isomorphic to a quotient of the $\hbar$-formal tensor algebra over $\mathbb{R}^n$ by a two-sided ideal $I_\hbar$. This conjecture was recently proved by D. Calaque, G. Felder and C. Rossi. In this talk, we shall give a simple interpretation and generalization to non-polynomial Poisson structures of this fact in terms of the Baker-Campbell-Hausdorff formula. The formula obtained gives new insights on the presence of graphs with oriented cycles in Kontsevich's star-product.

Alejandro Uribe, University of Michigan

Title: The spectral function of a Riemannian orbifold.

Abstract: Hörmander's theorem on the asymptotics of the spectral function of an elliptic operator is extended to the setting of orbifolds. In contrast with the manifold case, the asymptotics depend on the isotropy type of the point at which the spectral function is computed. There is a full "trace formula", which involves a regularization of the Hamilton flow in the cotangent orbi-bundle of the orbifold. This is joint work with Elizabeth Stanhope.

Martin Bordemann, Université de Haute Alsace

Title: Classical BRST cohomology for general graded commutative algebras

Abstract: Let A be graded commmutative unital Poisson algebra over a graded commutative ring containing the rationals, let I be a graded commutative ideal of A which is a Poisson subalgebra (coisotropic ideal), and let B be the graded commutative algebra A/I. Out of a graded Koszul-Tate resolution $S_A V \to B \to 0$ of B as a graded A-module and a generalized connection in the graded projective A-module V we construct a graded Poisson bracket on $\mathrm{Hom}_A(S_A V, S_A V)$ which deforms the Lecomte-Roger Big Bracket by the Poisson bracket on A and some curvature terms (Rothstein bracket), and an odd element θ in this Hom-space (the so-called BRST charge) such that the Poisson bracket with θ defines a cohomology on the Hom space which contains the Poisson reduced algebra $A_{red} = N(I)/I$ as a Poisson subalgebra, where $N(I)$ is the Lie idealizer of I in A. The main techniques come from the Hopf algebra structure of $S_A V$, and from derived brackets. Our approach makes it possible to treat Poisson reduction in differential geometry and in algebraic geometry on equal footing, but does not need any hypothesis on regularity or finitely many generators: so it may also be applied in singular situations or in field theory (?).

Mark Gotay, University of Hawaii

Title: Quantization via Prequantization

Abstract: A few years ago Kostant observed that classical mechanics on a symplectization of a prequantum circle bundle over a phase space X amounts to prequantized mechanics on the original phase space. Here, in the same vein, we explain how a polarized 'quantum product' on a prequantum circle bundle provides a *genuine* quantization of X. This is joint work with Christian Duval (Marseille).

Eugene Lerman, University of Illinois at Urbana-Champaign

Title: Oribifolds and (pre)quantization.

Abstract: (this is joint work with Anton Malkin) If one regards orbifolds as Lie groupoids one can define a functor that attaches to a Lie groupoid with a certain extra data ("integral 2-form") a Hermitian line bundle with a connection over the groupoid. In the case where our orbifold is a point with an action of the finite group the functor gives 1-dimensional representations.

Victor Palamodov, Tel Aviv University

Title: Associative deformation and quantization in analytic geometry

Abstract: The category of complex analytic spaces is a model for the theory of quantization of singular spaces that looks similar to the classical theory of deformations of analytic spaces (Kodaira-Spencer-Grothendieck). The notion of associative deformation of a complex analytic spaces includes both concepts at least on the formal level. The basic tool for the formal theory is the analytic Hochschild cohomology of the structure sheaf which plays the same role as the sheaf of tangent fields in the deformation theory. Any formal associative deformation of an arbitrary analytic space is canonically isomorphic at the linear term level to the direct sum of a (commutative) deformation and a quantization (which can be considered as a skew-commutative deformation). Unlike the smooth case there is no canonical splitting at the next steps of a infintesimal extension. This means that one can not canonically separate quantizations and deformations in the category of singular analytic spaces.

Gerald Schwarz, Brandeis University

Title: Real double coset spaces and their invariants:

Abstract: Let G be a real form of a complex reductive group. Suppose that we are given involutions σ and θ of G. Let $H = G^\sigma$ denote the fixed group of σ and let $K = G^\theta$ denote the fixed group of θ. We are interested in calculating the double coset space $H\backslash G/K$. We use moment map and invariant theoretic techniques to calculate the double cosets, especially the ones that are closed. One salient point of our results is a stratification of a quotient of a compact torus over which the closed double cosets fiber as a collection of trivial bundles.

Markus Pflaum, University of Colorado at Boulder

Title: On the geometry and topology of orbit spaces of proper Lie groupoids

Abstract: In the talk a natural stratification of the orbit space of a proper Lie groupoid will be constructed by using the slice theorem by Weinstein and Zung for proper Lie groupoids and an extension of it. Next it is shown that one can construct invariant riemannian metrics for proper Lie groupoids and that this induces the structure of a singular riemannian foliation on the object space of the groupoid. The orbit space then inherits a natural length space structure whose properties will be studied. In particular I will show that the orbit space with this metric is locally geodesically convex, a property which has crucial implications for Cech and de Rham cohomology. Finally, I will consider applications of these results to the construction of atlas functors for proper Lie groupoids. The talk is on joint work with H. Posthuma and X. Tang.

Klaas Landsman, Radboud University Nijmegen

Title: Functoriality of quantization: an operator-algebraic approach

Abstract: It is an old dream that quantization is functorial from some category of classical data to some category of quantum data. I proposed a specific way operator-algebraic way of doing this in 2003, and showed that the "quantization commutes with reduction" conjecture of Guillemin and Sternberg then becomes a special case of the hypothetical functoriality of quantization (arXiv:math-ph/ 0307059). Because of this, functoriality would give a systematic method for quantizing singular symplectic quotients. In fact, whilst the Guillemin-Sternberg conjecture as originally formulated only made sense for compact groups acting on compact spaces, the operator- algebraic setting seamlessly generalizes the conjecture to the noncompact case, where it essentially becomes a problem in equivariant index theory akin to the Baum-Connes conjecture in noncommutative geometry. With my PhD student Peter Hochs, I proved a special case of the ensuing "noncompact" Guillemin-Sternberg conjecture [J. of K- Theory 1, 473-533 (2008), arXiv:math-ph/0512022], and more recently it was proved in great generality by Varghese Mathai and Weiping Zhang [Adv. Math. 225, 1224-1247 (2010), arXiv:0806.3138]. The aim of the talk is to give an overview of this entire development.

Rui Loja Fernandes, Instituto Superior Tecnico Lisbon

Title: The Equivariant Picard group in Poisson geometry

Abstract: I will discuss a version of the Picard group of a Poisson manifold in the presence of symmetry, i.e., when a Poisson action is around. This is an essential step to understand the corresponding problem of Morita equivalence of algebras obtained by deformation quantization under the presence of symmetry.

Hans-Christian Herbig, QGM Aarhus University

Title: On Deformations of Singular Poisson Algebras

Abstract: We present several versions of a constructive theorem on associative deformations of singular Poisson algebras arising in Hamiltonian reduction. The results are obtained by using the simplest incarnation of the Batalin-Fradkin-Vilkovisky (BFV)- method, the so-called first order formalism. The main input is here, that the Koszul complex on the moment map is a resolution of the algebra of smooth (or regular) functions on the zero fibre of the moment map. We classify unitary representations of tori and SU(2) having this property. We set the results into some broader perspective. The results are an outcome of a long-term collaboration with Martin Bordemann, Markus Pflaum and Srikanth Iyengar.

Marius Crainic, Utrecht University

Title: Prequantization and the integrability of Lie brackets

Abstract: It is well known that a closed 2-form is prequantizable if and only if it is integral. On the other hand, associated to a closed 2-form there is an extension of the tangent bundle by the reals which is a Lie algebroid; and it turns out that the integrability of this Lie algebroid is equivalent to the condition that the group of spherical periods of the 2-form is a multiple of the integers. In this talk I will describe the direct relationship between the prequantization and integrability. This will be carried out in the general context of multiplicative 2-forms on groupoids. Since groupoids often appear as "desingularizations" of singular spaces, this framework may be useful for geometric quantization schemes of singular spaces.

Vladimir Fock, IRMA Strasbourg

Title: Partial compactification of cluster varieties and quantization

Abstract: Cluster varieties are smooth affine algebraic varieties constructed combinatorially and admitting quantization. Many examples of such varieties are actually open dense subsets of projective or quasi-projective possibly algebraic varieties which also require quantization. The main example of this case is the moduli space of complex flat connections on Riemann surfaces or the complex Lie groups. In the talk we describe a class of partial compactifications of cluster varieties compatible in a certain sense with cluster structure as well as some features of their quantization.

Martin Schlichemaier, Université de Luxembourg

Title: Berezin symbols and Berezin transformations revisited

Abstract: In the context of Kaehler manifolds the Berezin Toeplitz quantization is a naturally adapted quantization scheme. Related to it is the theory of Berezin symbols and the associated Berezin transform. As an intermediate object Bergman kernels appear. We will start with arbitrary quantizable Kaehler manifolds. But to obtain precise results we have to concentrate on the compact case. This is partly joint work with Alexander Karabegov.