English version

Topological Quantum Field Theories and Moduli Spaces

Level of course
PhD Course

Semester/quarter
3rd + 4th quarter (Spring 2011)

Hours per week
4

Name of lecturer
Jørgen Ellegaard Andersen and Johan Martens

Objectives of the course
Since its introduction in the late 1980s, the concept of a Topological Quantum Field Theory (TQFT) has become a central notion in geometry, topology and representation theory. Roughly speaking, an n-dimensional TQFT is an apparatus for assigning a vector space to each (n-1)-dimensional manifold, and an element in the vector space of the boundary to each n-manifold with boundary. A cobordism between two (n-1) manifold becomes an operator between the associated vector spaces, which one can interpret as a form of time-evolution between two spaces of states.

Motivated by the formal structures of physical quantum field theories, this concept of a TQFT offers a framework to organize, understand and unify a lot constructions in knot theory and low-dimensional topology, as well as a way of obtaining new insights and results. Various TQFTs exist, and as with cohomology, there are often several ways one can mathematically construct these.

In this course we shall introduce the general notion of a TQFT as well as discuss some TQFTs in detail, in particular focusing on constructions involving the quantization of moduli spaces.

Course contents
1) General introduction: motivation, history, Atiyah-Segal axioms.

2) Chern-Simons theory: physical motivation through path-integral, link with Jones polynomials, rigorous construction through quantization of moduli spaces of bundles.

3) Donaldson theory as a TQFT.

4) The geometric Langlands program as an equivalence of TQFTs.

Learning outcomes and competences
At the end of the course the student should have a solid understanding of the general concept of a topological quantum field theory, as well as be familiar with the main examples used in current research.

Literature
Relevant books and research papers.

Teaching methods
Four hours of lectures per week

Assessment methods
Passed / not passed will be based on the students participation in the course

Credits
10 ECTS

Language of instruction
English

Course enrolment
Please send an e-mail to Maiken Kirdorf Nielsen, maiken@imf.au.dk