English version

Introduction to gauge theory

Level of course
PhD Course

Semester/quarter
1st + 2nd quarter (Autumn 2011)

Hours per week
4

Name of lecturer
Jørgen Ellegaard Andersen, Robert Penner, and Kim Frøyshov

Objectives of the course
In physics, a gauge theory is a type of field theory which involves what in mathematics is known as a connection. For instance, in electromagnetism the electromagnetic potential is a U(1)–connection over space-time. In the 1980's Donaldson applied a certain gauge theory (non-Abelian Yang–Mills theory) to obtain several ground-breaking results about smooth 4–manifolds. Since then, gauge theory has played an essential role in 3– and 4–dimensional topology.

The goal of this course is to explain how moduli spaces of solutions to certain gauge-theoretic equations (the instanton equation studied by Donaldson, and the monopole equations discovered later by Seiberg and Witten) can be used to define numerical invariants of closed oriented smooth 4–manifolds. However, a large part of the course will be occupied with the necessary background in differential geometry.

Since this course will not go into the analytical aspects, the discussion of the Hodge theorem and many results in gauge theory will be rather informal. If there is sufficient interest, a continuation of this course in the following semester will provide some background in analysis (i.e. Sobolev spaces and elliptic operator theory) and also discuss aspects of Floer homology.

Course contents

- De Rham cohomology, elliptic complexes, and the Hodge theorem
- Connections in vector bundles and in principal bundles
- Chern–Weil theory
- Donaldson invariants
- Clifford algebras and Dirac operators
- Seiberg–Witten invariants

Literature

S. Kobayashi and K. Nomizu: *Foundations of Differential Geometry*

H. Lawson and M.-L. Michelsohn: *Spin Geometry*

S. K. Donaldson and P. B. Kronheimer: *The Geometry of Four-Manifolds*

J. W. Morgan: *The Seiberg-Witten equations and applications to the topology of smooth four-manifolds*

Teaching methods

4 hours of lectures per week

Assessment methods

Passed / not passed will be based on the students participation in the course

Credits

10 ECTS

Language of instruction

English