Introduction to Lagrangian Floer homology

Level of course
PhD Course

Semester/quarter
1st + 2nd quarter (Autumn 2010)

Hours per week
4

Name of lecturer
Reza Rezazadegan

Objectives of the course
Lagrangian Floer homology has many applications in symplectic geometry, in construction of topological invariants and in homological mirror symmetry. Besides it is an important example of infinite dimensional Morse theory and of the theory of pseudoholomorphic curves. The purpose of this course is to introduce the students to the sympletic geometry, Morse theory and analysis relevant to Lagrangian Floer homology.

Prerequisites
Basic differential geometry: DeRham cohomology, Lie derivative, covariant derivative, etc. Basic algebraic topology: fundamental group, π_2, covering spaces, homology and cohomology. Basic notions of functional analysis such as L^p spaces and operator norm. Familiarity with Sobolev spaces is helpful but is not assumed.

Course contents

1. Basic symplectic geometry
    - Symplectic preliminaries
    - Almost complex structures
    - Lagrangian correspondences
2. Morse homology
    - Morse homology in finite dimensions
    - Morse theory of the action functional: pseudoholomorphic curves

3. Pseudoholomorphic curves
    - Pseudoholomorphic polygons
    - Gromov compactness and Gromov topology
4. More symplectic geometry
    - Maslov index
    - Monotonicity
5. Construction of Lagrangian Floer homology
    - Sobolev spaces and elliptic operators
    - Transversality
    - Gluing of pseudoholomorphic polygons
    - Definition of Lagrangian Floer cohomology
    - Grading
    - Orientation of the moduli spaces
    - Floer cohomology for Lagrangian correspondences
6. Properties [time permitting]
    - Morphisms between Floer groups: quilts
    - Mapping cones
    - Functoriality

Literature
Unfortunately there is no text covering the topic of the course. An essay on the subject will be provided by myself. A theory with a similar construction but with different applications is developed in the following lecture note which is available at http://www.math.ethz.ch/~salamon/publications.html.

- D. Salamon: *Lectures on Floer homology*, Lecture Notes for the IAS/PCMI Graduate Summer School on Symplectic Geometry and Topology

The general theory of pseudoholomorphic curves is studied in

- D. McDuff, D.A. Salamon: *J-Holomorphic Curves and Symplectic Topology*, AMS Colloquium Publications, Vol. 52, 2004.

For an introduction to symplectic geometry see

- Robert Bryant: *An introduction to Lie groups and symplectic geometry*, in Geometry and quantum field theory (Park City, UT, 1991), IAS/Park City Mathematics, vol. 1 (1995), pp. 5-181,Amer. Math. Soc., Providence, RI.

Teaching methods
4 hours of lectures per week

Assessment methods
Passed / not passed will be based on the students participation in the course and the assigned homework

Credits
10 ECTS

Language of instruction
English

Course enrolment
Please send an e-mail to Maiken Nielsen, maiken@imf.au.dk