Introduction to Lie groups and Chern-Simons theory

Fall 2009

Level of course

PhD Course

Semester/quarter

1st + 2nd quarter (Fall 2009)

Hours per week

4

Name of lecturer

Jørgen Ellegaard Andersen, Benjamin Himpel

Prerequisites

A basic understanding of differentiable manifolds and differential forms.

Objectives of the course

This course is a PhD course in an active field of geometric topology at the border to mathematical physics. Witten introduced classical Chern-Simons theory to topology 20 years ago, when he defined invariants for knots in 3-manifolds by an integral over a certain infinite-dimensional space, which up to today have not been entirely understood. However, they motivated lots of interesting questions and results in knot theory and low-dimensional topology, as well as the development of entirely new fields.

After an introduction to Lie groups, Lie algebras, principal bundles, connections and gauge transformations, we will carefully construct the Chern-Simons action and study the moduli space of its classical solutions. This yields Taubes' beautiful and influential description of Casson's invariant for homology 3-spheres, for which we need such concepts as differential operators and spectral flow. This naturally leads to subjects like the eta invariant and the rho invariant on the one hand as well as the quantization of the Chern-Simons action and Witten's invariants on the other.

Course contents

- Lie groups and Lie algebras
- Algebraic sets and the Zariski tangent space
- Connections in principal bundles, curvature, and Chern-Weil theory
- The Chern-Simons function
- Casson's invariant and gauge theory
- Classical Chern-Simons theory

Learning outcomes and competences

At the end of the course the student will have a solid understanding of the theory of lie groups and connections in principal bundles as well as other prerequisites for studying gauge theory. Furthermore, he will be familiar with the gauge theoretic version of Casson's invariant and classical Chern-Simons theory.

Literature

- S K Donaldson, P B Kronheimer, The geometry of four-manifolds, Oxford Mathematical Monographs, The Clarendon Press Oxford University Press, New York (1990), Oxford Science Publications.
- D S Freed, Classical Chern-Simons theory. I, Adv. Math. 113 (1995) 237303.
- S Hu, Lecture notes on Chern-Simons-Witten theory, World Scientific Publishing Co. Inc., River Edge, NJ (2001). With a preface by E. Witten.
- F W Warner, Foundations of differentiable manifolds and Lie groups, volume 94 of Graduate Texts in Mathematics, Springer-Verlag, New York (1983). Corrected reprint of the 1971 edition.

Teaching methods

4 hours of lectures per week

Assessment methods

Passed / not passed will be based on the students participation in the course

Credits

10 ECTS

Language of instruction

English