Sporadic & Exceptional

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幾何、 量子拓撲 與 漸進分析

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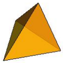
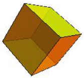
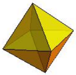
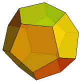
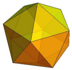
Classification Problems: regulars vs. exceptionals

- regular solids/tessellations: infinite families of shapes and a few special ones
  - e.g. regular polygons vs. Platonics, prisms vs. Archimedeans
  - tessellation of Riemann surfaces
- Lie (semi-simple) algebras: classical $(ABCD)_n$ vs. exceptionals EFG
- Finite (simple) groups: Lie groups over finite fields vs. Sporadics
- Finite-type quiver representations: $(AD)_n$ vs. E
- 2D CFT: modular-invariant partition functions are ADE
- Modular curves $\Gamma_0(N)\backslash\mathcal{H}$: genus 0 means N one of 15 values
- *etc.* ...
- **LESSON:** examine the *specialness* of exceptionals, wealth of structures, possibly INTERWOVEN $\rightsquigarrow$ Exceptionology

McKay Correspondence, ADE-ology

- Platonic Solids:

regular polygons	Tetrahedron	Cube & Octahedron	Dodecahedron & Icosahedron
$P_{n\geq 3}$		 	 

- Sym in $SO(3)$: [Cyclic] $\mathbb{Z}/n\mathbb{Z}$, [Dihedral] D_n, [T] A_4, [C/O] S_4, [D/I] A_5
- Embed in $SU(2)$: $0 \to \mathbb{Z}/2\mathbb{Z} \to SU(2) \to SO(3) \to 0$

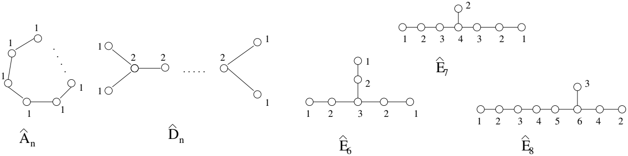
G	$\hat{A}_n$	$\hat{D}_n$	$\hat{T}$	$\hat{O}$	$\hat{I}$
$\lvert G\rvert$	n	$4n$	24	48	120

ADE pattern

- McKay [1980]: places in a concrete setting, take defining $\mathbf{2}$ of G and form multiplicity decomposition over irreps $\mathbf{r}_i$

$$\mathbf{2} \otimes \mathbf{r}_i = \bigoplus_{j=1}^{n} a_{ij} \mathbf{r}_j \ , \quad n = \#Conj(G) = \#Irrep(G)$$

- $a_{ij} = 2\mathbb{I} -$ Cartan mat. of affine ADE! i.e., a_{ij} is adjacency mat of quivers



$r_i = \dim \mathbf{r}_i =$ Dynkin labels (dual Coxeter numbers), in particular $\sum_i r_i^2 = |G|$

- Algebro-geometrically: $\mathbb{C}^2/(G \subset SU(2)) \simeq$ Local K3
  - use to construct gauge theories from D-branes [Douglas-Moore, '96]
  - $\mathbb{C}^3/(G \subset SU(3)) \simeq$ Local Calabi Yau 3 [Hanany-YHH, '98]

Trinities

- Classical Enumerative Geometry
  - Cayley-Salmon, 1849: 27 lines on cubic surface $[\mathbb{P}^3|3]$
  - Jacobi, 1850: 28 bitangents of quartic curve $[\mathbb{P}^2|4]$
  - Clebsch, 1863: 120 tritangent planes of sextic curve $[\mathbb{P}^4|1,2,3]$
- Recall dim of fundamental representations (cf. Hitchin [2000] Clay Lecture)

$$\begin{aligned} \dim_F(E_6) &= 27 \ , \\ \dim_F(E_7) &= 56 = 28 \times 2 \ , \\ \dim_F(E_8) &= 248 = 120 \times 2 + 8 \ . \end{aligned}$$

- Rmk: Arnold [1980s]: $\mathbb{R}, \mathbb{C}, \mathbb{H} \sim E_{6,7,8}$ a unified scheme (?)
  - $PSL(n,p) \curvearrowright \mathbb{P}^n(\mathbb{F}_p)$ non-trivially on only p points (out of $\frac{p^n-1}{p-1}$) and simple iff $p = 5, 7, 11$, when $\simeq A_4 \times \mathbb{Z}_5, S_4 \times \mathbb{Z}_7, A_5 \times \mathbb{Z}_{11}$; $(5,7,11) = 2r+3$;
  - #edges of $(T, O, I) = (2 \cdot 3, 3 \cdot 4, 5 \cdot 6) = (r+1)(r+2)$; $r = \dim_{\mathbb{R}}(\mathbb{R}, \mathbb{C}, \mathbb{H})$

A Geometric Framework: del Pezzo Surfaces $E_{n=0...8}$

- dP_n: $\mathbb{P}^2$ blown up at up to $n = 8$ generic points is surface of deg $= 9 - n$
  - intersection matrix of curve classes $H^2(dP_n, \mathbb{Z})$ is Cartan matrix of affine E_n

    (rmk: $E_{5,4,3,2,1} \simeq (D_5, D_4, A_2 \times A_1, A_2, A_1)$)

cubic surface $[3\|3]$ is dP_6	:	with 27 (-1)-curves
$dP_7 \to \mathbb{P}^2$ branched on $[2\|4]$	:	56 (-1)-curves pair to 28 bitangents
$dP_8 \to \mathbb{P}^2$ branched on $[4\|1,2,3]$	:	240 (-1)-curves pair to 120 tritangent planes

  - $\#(-1)$curves $=$ Rank(Mori cone of effective curve classes)

- also recall: $\#$ bitangents to genus g curve $= 2^{g-1}(2^g - 1)$

  $g([2|4]) = 3, \qquad g([4|1,2,3]) = 4$

- Rmk: Fermat model of $[4|1,2,3]$ is Bring's sextic

  $\mathcal{B} = \{\sum_i x_i^3 = \sum_i x_i^2 = \sum_i x_i = 0\} \subset \mathbb{P}^4$

Sporadic Simple Groups

- Classification of finite simple groups complete (from Galois to circa. 2008):
  - infinite families: $\mathbb{Z}/p\mathbb{Z}$, $A_{n\geq 5}$, Lie groups over $\mathbb{F}_q$
  - 26 sporadics (exceptionals)
- largest 3 sporadics are

Name	Notation	Order
Monster	$\mathbb{M}$, F_1	$2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71 \sim 10^{54}$
Baby Monster	$\mathbb{B}$, F_2	$2^{41} \cdot 3^{13} \cdot 5^6 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 31 \cdot 47 \sim 10^{33}$
Fischer 24'	Fi'_{24}, F_{3+}	$2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29 \sim 10^{24}$

- Monster: 194 conjugacy classes/irreps of degree (linear character table 194×194) $\quad r_i = \{1, 196883, 21296876, 842609326, 18538750076, \ldots\}$
- McKay, 1978 observation: $196883 + 1 = 196884$

Monstrous Moonshine

- absolute invariant (attributed to Klein, but known earlier) j-function
  - meromorphic on upper half-plane $\mathcal{H} \ni z$ and $j(\gamma \cdot z) = j(\frac{az+b}{cz+d}) = j(z)$, $\gamma \in \Gamma := PSL(2;\mathbb{Z})$
  - unique: all invariants $= \mathbb{Q}(j) \rightsquigarrow$ Hauptmodul or principal modulus
  - "weight 0 modular form", but pole at $i\infty$, Fourier series (*nome* $q := \exp(\pi i z)$)

$$j(q) = \frac{1}{q} + 744 + 196884q + 21493760q^2 + 864299970q^3 + \ldots$$

  - $j(z) = 1728\frac{g_2^3(z)}{\Delta(z)}$, with g_k Eisenstein series, $\Delta(z) = g_2^3 - 27g_3^2 = \eta(z)^{24}$
- McKay's observation goes on

$$\begin{aligned} 1 &= r_1 \\ 196884 &= r_1 + r_2 \\ 21493760 &= r_1 + r_2 + r_3 \\ 864299970 &= 2r_1 + 2r_2 + r_3 + r_4 \\ &\ldots \end{aligned}$$

THEOREM [Borcherds 1992]

[Proved Moonshine Conjecture: Conway-Norton, McKay-Thompson, Atkin-Fong-Smith, Frenkel-Lepowsky-Meurman] There exists an infinite-dim graded module $V = V_0 \oplus V_1 \oplus V_2 \oplus \ldots$ of $\mathbb{M}$ such that

- $V_0 = \rho_1$, $V_1 = \{0\}$, $V_2 = \rho_1 \oplus \rho_{196883}$, $V_3 = \rho_1 \oplus \rho_{196883} \oplus \rho_{21296876}$, $\ldots$
- for each conjugacy class g of $\mathbb{M}$, define McKay-Thompson series

  $$T_g(q) = q^{-1} \sum_{k=1}^{\infty} \mathrm{Ch}_{V_k}(g) q^k = q^{-1} + 0 + h_1(g)q + h_2(g)q^2 + \ldots$$

  - $T_{g=\mathbb{I}} = j(q)$;
  - $T_g(q)$ is (normalized) generator of a genus zero function field for a group G between $\Gamma_0(N)$ and its normalizer $\Gamma_0(N)^+$ in $PSL(2, \mathbb{R})$ (genus is that of **modular curve** $G \backslash \mathcal{H}$);
  - N s.t. $N/n = h \in \mathbb{Z}_{>0}$; $\quad h|24$, $h^2|N$ with $n = \mathrm{Order}(g)$
- $\ldots$

Remarks

- Proof used Vertex Operator Algebras from string theory/CFT
- congruence group $\Gamma_0(N) := \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid c \bmod N = 0\}$; normalizer $\Gamma_0(N)^+ := \{\frac{1}{\sqrt{e}}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2,\mathbb{R})\ , \mid a,b,c,d,e \in \mathbb{Z},\ ad - bc = e\ ,\ e|N,\ e|a,\ e|d,\ N|c\}$
- T_g is the Hauptmodul of group G st. $\Gamma_0(N) \subset G \subset \Gamma_0(N)^+$ *commensurate* with $\Gamma = PSL(2;\mathbb{Z})$ (i.e., $G \cap \Gamma$ is finite index in Γ and in G)
- **Jack Daniels Problem:** When N is prime p, genus$(\Gamma(p)^+) = 0$ iff p is one of the 15 monstrous primes [Ogg, 1974] , explanation OPEN?

  Also: p is the 15 supersingular primes for elliptics curves, i.e., over $\mathbb{F}_{p^r}$ same as over $\mathbb{F}_p$

The "Missing" Constant

- monstrous moonshine module $V_1 = \{0\}$, so constant is renormalized; however, McKay also noticed that 744 in $j(q)$ is 248×3
  - $j(q)^{\frac{1}{3}} = q^{-\frac{1}{3}} \left(1 + 248q + 4124q^2 + 34752q^3 + \ldots\right)$
  - $248 = 248 \ , 4124 = 3875 + 248 + 1 \ , 34752 = 30380 + 3875 + 2 \cdot 248 + 1 \ldots$
  - Kac [1978]: $j(q)^{\frac{1}{3}}$ is the character for the level-1 heightest-weight rep of $\hat{E}_8$
  - YHH-McKay '14: $j(q)^{\frac{1}{n}}$ for $n|24$ have integer coefs; $n = 1, 2, 3$ are McKay-Thompson series, only $n = 3$ has $\mathbb{Z}_{\geq 0}$ coefs, what about rest?
- perhaps not surprising: $j(z) = 1728 \frac{g_2^3(z)}{\Delta(z)}$ and $g_2 = \theta_{\Lambda(E_8)}(q) = \sum\limits_{x \in \Lambda(E_8)} q^{|x|^2/2} = 1 + 240 \sum\limits_{n=1}^{\infty} \sigma_3(n) q^n$ is the theta-series of root lattice of E_8 (1st non-trivial unimodular even lattice), 2 curiosities
  - explain the 240 in terms of the (-1) curves?
  - $\sigma_1(240) = 744$

A Pair of Trinities

- The modular function $j(q)$ knows about $\mathbb{M}$ AND E_8
- Big Picture for the largest 3 (NB., Schur multipliers are resp (1,2,3))?

Modularity

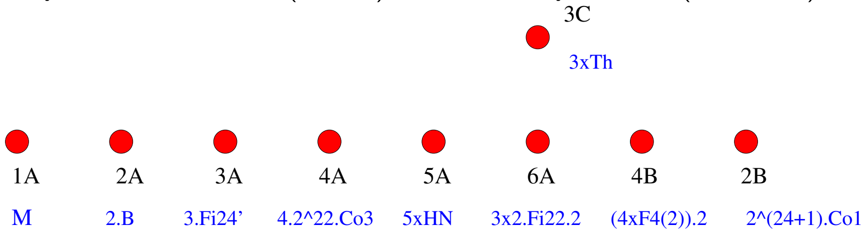
Sporadic Groups (Schur Multiplier)	Lie Algebras (affine $\mathbb{Z}_n$ symmetry)
$\mathbb{M}(1)$	$E_8(1)$
$\mathbb{B}(2)$	$E_7(2)$
$Fi'_{24}(3)$	$E_6(3)$

- More evidence [McKay, 1985]:
  - ATLAS notation: conjugacy class xN, x order and N capital letter indexing

Conj Class	$1A$	$2A$	$2B$	$3A$	$\ldots$
Centralizer	$\mathbb{M}$	$2.\mathbb{B}$	$2^{1+24}.Co_1$	$3.Fi'_{24}$	$\ldots$

$\mathbb{M}$ and E_8-Dynkin

- Only 2 involution classes (2A, 2B); 2A*2A $\rightsquigarrow$ only 9 classes (out of 194):



- orders of conjugacy classes are precisely E_8 Dynkin labels!
- OPEN: no explanation, especially concept of adjacency
- amazing that a group as large as $\mathbb{M}$ multiplies to only up to order 6: $\mathbb{M}$ is a 6-transposition group

Pattern persists

- Baby is 4-transposition and Fischer is 3-transposition:

$\mathbb{B}$	Fi'_{24}
$\begin{array}{ccccccccccccc} & & & & & & 2c & & & & & & \\ & & & & & & \vert & & & & & & \\ 1a & - & 2b & - & 3a & - & 4b & - & 3a & - & 2b & - & 1a \end{array}$ $\hat{E}_7$	$\begin{array}{ccccccccc} & & & & 1a & & & & \\ & & & & \vert & & & & \\ & & & & 2a & & & & \\ & & & & \vert & & & & \\ 1a & - & 2b & - & 3a & - & 2a & - & 1a \end{array}$ $\hat{E}_6$
$1 - 2 - 3 = 4 - 2$ $\hat{F}_4$	$1 - 2 \equiv 3$ $\hat{G}_2$

- Höhn-Lam-Yamaguchi, 2010, persists for our pair of trinities; and constructed the VOA/moonshine modules for them (cf. Queen, Duncan, Gannon)

$(\mathbb{M}, \mathbb{B}, Fi'_{24})$ versus $E_{8,7,6}$

- A geometric/modular setting [YHH-McKay, '15]?
- Cusp Numbers
  - Cusps for any congruence subgroup $G \subset \Gamma := PSL(2;\mathbb{Z})$: finite set of Γ-orbits in $\mathbb{Q} \cup \infty$; Def cusp number $= |C(G)|$
  - Rmk: need to add cusp when forming modular curve $X(G) \simeq G \backslash \mathcal{H}$
  - $|C(\Gamma)| = 1$ since $C(PSL(2;\mathbb{Z})) = \{\infty\}$;
  - recall $\Gamma_0(N)$, $|C(\Gamma_0(N))| = \sum\limits_{d|N, d>0} \phi(\gcd(d, N/d))$
  - need more sophistication: $|C(G)|$ for moonshine group $\Gamma_0(N) \subset G \subset \Gamma_0(N)^+$
- Conway-Norton, 1979: computed all McKay-Thomson series and much info (e.g. cusps) for their associated G
- Norton, Cummins et al: created a database over the years

Moonshine Groups in more Detail

- All moonshine groups for $\mathbb{M}$ (i.e., the genus 0 modular groups for McKay-Thompson series) are of the form $\langle \Gamma_0(n|h), w_{e_1}, w_{e_2}, \ldots \rangle$ for Hall divisors e_i of n/h; $N = nh^2$
  - Define Atkin-Lehner involution $w_e = \frac{1}{\sqrt{e}} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & 0 \\ 0 & 1 \end{pmatrix}$ with $e||N$ (Hall divisor, $e|N$ and $\gcd(e, h := \frac{N}{e}) = 1$), $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(h)$, $d \equiv 0 (\mathrm{mod}\ e)$
  - Define $F_h := \begin{pmatrix} h & 0 \\ 0 & 1 \end{pmatrix}$, $\Gamma_0(n|h) := F_h^{-1} \Gamma_0(\frac{n}{h}) F_h$ and $w_e := F_h^{-1} W_e F_h$
- NB. Of the 194 conjugacy classes of $\mathbb{M}$, some are just in Galois orbits, $\rightsquigarrow$ 172 classes (i.e., rational character table is 172×172)
  - Rmk: Thus 172 distinct McKay-Thompson
  - Rmk: There are replication formulae (functional equations) reducing 172 further to 163 (WHY largest Heegner number?)

Explicit Examples

Class	McKay-Thompson	G	$\lvert C(G)\rvert$
$1A$	$1728\frac{g_2^3(z)}{\eta(z)^{24}} - 744 = j_M(q)$	$PSL(2;\mathbb{Z})$	1
$2A$	$\left(\left(\frac{\eta(q)}{\eta(q^2)}\right)^{12} + 2^6\left(\frac{\eta(q^2)}{\eta(q)}\right)^{12}\right)^2 - 104$ $= q^{-1} + 4372q + 96256q^2 + 1240002q^3 + \ldots$	$\langle\Gamma_0(2), w_1, w_2\rangle$	1
$2B$	$24 + \frac{\eta(q)^{24}}{\eta(q^2)^{24}}$ $= q^{-1} + 276q - 2048q^2 + 11202q^3 + \ldots$	$\Gamma_0(2)$	2
$3C$	$q^{\frac{1}{3}}\left(\frac{\eta(q)}{\eta(q^2)^8} + 256\frac{\eta(q^2)}{\eta(q)}\right)^{16}$ $= q^{-1} + 248q + 4124q^2 + 34752q^3 + \ldots$	$\langle\Gamma_0(3), w_3\rangle$	1

Monstrous Cusps: $\mathbb{M}$ and E_8

- Take the 172 (rational) classes (Galois conjugates have same cusps, McKay-Thompson, etc);

$$(\text{Class Name}, |\text{Cusp}|) =$$
$$\{(1A,1),(2A,1),(2B,2),(3A,1),(3B,2),\ldots,(120A,1),(119AB,1)\}$$

  bin count of cusp numbers: $(1^{60}, 2^{75}, 3^{12}, 4^{20}, 6^3, 8^2)$

- Observation: $\sum_g C_g(\mathbb{M}) = 360 = 3 \cdot 120$, $\sum_g C_g(\mathbb{M})^2 = 1024 = 2^{10}$
- recall: 120 is the #tritangents on sextic (or 240 (-1)curves on dP_8)
- next in the family of Bring's sextic, genus 4 curve is Fricke's octavic, genus 9 curve $\mathcal{F} = \{\sum_i x_i^4 = \sum_i x_i^2 = \sum_i x_i = 0\} \subset \mathbb{P}^4$ has 2.2^{10} bitangents

Baby Moonshine

- Class $2A$ of $\mathbb{M}$ has $2.\mathbb{B}$ (double cover of Baby) as centralizer
  - dim(irreps($2.\mathbb{B}$)) = 1, 4371, 96255, 1139374 ..., cf. T_{2A}
  - 247 conjugacy classes (minus conjugates) $\rightsquigarrow$ 207 distinct McKay-Thompson
  - Höhn 2007 Moonshine for $2.\mathbb{B}$ (explicit VOA and M-T)
  - some M-T coincide with $\mathbb{M}$ M-T, some are new
- Cummins, Ford, McKay, Mahler, Norton, 1990s M-T belong to a class of so-called replicable functions
  - Recall (classical): $\sum\limits_{ad=n, 0 \le b < d} j(\frac{a\tau+b}{d}) = \text{Poly}_n(j(\tau))$
  - analogue for all M-T $T(q) \sim q^{-1} + \sum\limits_k b_k q^k$ satisfying the replication formulae
  - 616 of these, of genus 0; Notation [Norton] (n number x letter)

$$nX\text{(monstrous)}, \quad nx, \quad n\tilde{}x$$

Baby and E_7

- The 207 McKay-Thompson series for $2.\mathbb{B}$ and associated cusp numbers

  (conjugacy class [Atlas notation], M-T [Replicable notation], $|C(G)|$) =

  $(1a, 2A, 1),\ (2a, 4\tilde{}b, 1),\ (2b, 2a, 1), \ldots,\ (104b, 208\tilde{}a, 1), (110a, 220\tilde{}b, 1)$

- cusp numbers are $(1^{82}, 2^{61}, 3^{30}, 4^{25}, 6^9)$
- Observation: $\sum_g C_g(2.\mathbb{B}) = 448 = 2^3 \cdot 56$, $\sum_g C_g^2(2.\mathbb{B}) = 1320$
- Recall: $\dim_F(E_7) = \#(-1)$curves on $dP_7 = 56 \rightsquigarrow$

  #bitangents on $[2|4]$ = 28

Fischer and E_6

- Class $3A$ of $\mathbb{M}$ has $3.Fi'_{24}$ (triple cover of Fischer) as centralizer
  - dim(irreps$(3.Fi'_{24})$) = 1, 8671, 57477 ..., cf. T_{3A}
  - 265 conjugacy classes (minus conjugates) $\rightsquigarrow$ 213 distinct M-T
  - Matias 2014: explicit VOA and M-T

  (conjugacy class [Atlas notation], M-T [Replicable notation], $|C(G)|$) =

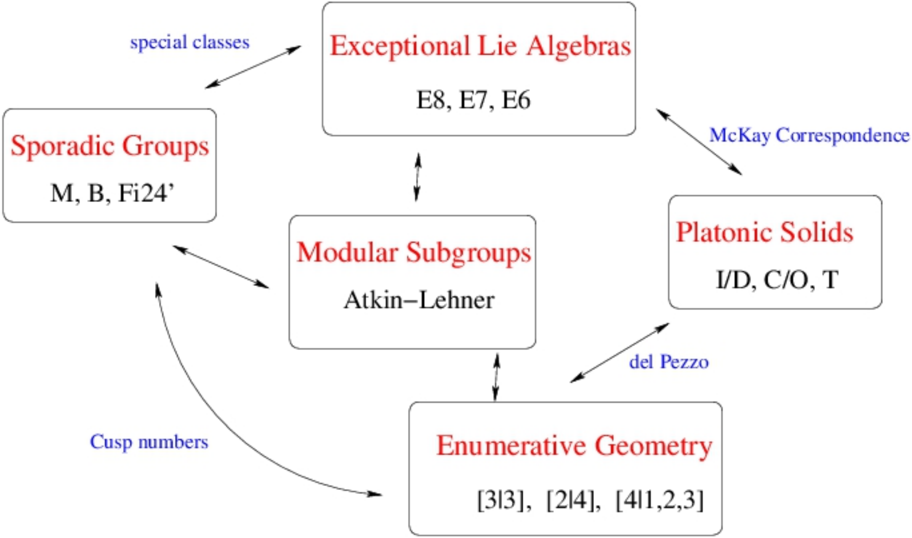
  $(1a, 3A, 1),\ (2a, 6A, 1),\ (2b, 6C, 2), \ldots (45d, 45a, 1), (60e, 60c, 1)$
- Observation:

  $\sum_g C_g(3.Fi'_{24}) = 440 = 2^4 \cdot 27 + 8(?)$, $\qquad \sum_g C_g^2(3.Fi'_{24}) = 1290$
- Recall: $\dim_F(E_6)$ = #lines on $dP_6 = 27$
- curious $+8$ in reverse to $\dim_F(E_8) = 248 = 120 \times 2 + 8$

Cusp Character

- Rmk: Geometric perspective on moonshine? Witten, Hirzebruch
  - 24-d spin manifold w/ elliptic (Witten) genus $j(q)$ & $\mathbb{M}$ action
  - Hopkins-Mahowald, 1998 found a manifold with all prop. except action of $\mathbb{M}$
- Thm: [YHH-McKay] For $\mathbb{M}, 2.\mathbb{B}, 3.Fi'_{24}$, $\exists$ a "cusp representation", i.e., weighted centralizer rep: take $v_\gamma = C_\gamma |Z(c_\gamma)| = C_\gamma \frac{|G|}{|c_\gamma|}$
- RMK: From representation theory point of view cusps are interesting; call it "cusp representation"; knows about the geometry/modularity. SKETCH PF:
  - any rep $R = \bigoplus_{i=1}^n R_i^{\oplus a_i}$ for irreps R_i of finite group $\rightsquigarrow$ multiplicities $a_k = \frac{1}{|G|} \sum\limits_{\gamma=1}^{n} \chi(R)\chi_k(c_\gamma)|c_\gamma|$ for conj class c_γ;
  - for centralizer rep: take $\chi(R) = |G|/|c_\gamma| \rightsquigarrow a_j = \sum\limits_{\gamma=1}^{n} \chi_j(c_\gamma) \in \mathbb{Z}_{\geq 0}$;
  - centralizers weighted with cusp numbers (beyond the group theory), no reason should be a character (could be virtual char). But, explicitly check all $a_j = \sum\limits_{\gamma=1}^{n} \chi_j(c_\gamma) C_\gamma \in \mathbb{Z}_{\geq 0}$

Summary



Exceptional Lie Algebras
E8, E7, E6
special classes
Sporadic Groups
M, B, Fi24'
McKay Correspondence
Modular Subgroups
Atkin–Lehner
Platonic Solids
I/D, C/O, T
del Pezzo
Cusp numbers
Enumerative Geometry
[3|3], [2|4], [4|1,2,3]

The Dangers of Moonshine

Brit facing 360 lashes in Saudi Arabia after being caught with homemade wine

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