CENTRE FOR QUANTUM GEOMETRY OF MODULI SPACES
DEPARTMENT OF MATHEMATICS
AARHUS UNIVERSITY

Programme: QGM Nielsen Retreat 2017

Friday 17 November

18:00 Dinner
20:00 Cheese & wine in the Library

Saturday 18 November (Auditorium: "Stalden")

08:00-09:00 Breakfast (each morning at the same time)
09:30-10:30 **Gaëtan Borot** *Geometric recursion*
10:45-11:45 **Gergely Bérczi** *Non-reductive moduli spaces and their cohomology*
12:00-13:00 Lunch
13:15-14:15 **Tobias Kildetoft** *An introduction to 2-categories and categorification*
14:30-15:30 **Martin de Borbon/Cristiano Spotti** *Singular KE metrics, "stable" tangent cones of algebraic varieties and applications*
15:45-16:45 **Martin de Borbon/Cristiano Spotti** *Singular KE metrics, "stable" tangent cones of algebraic varieties and applications*
18:00 Dinner
19:30-20:00 **Paolo Masulli** *The topology of neuronal networks and their activation patterns* (Auditorium: "Stalden")
20:00 Cheese & wine in the Library

Sunday 19 November

'JUNIOR SEMINAR DAY' – students for students (Auditorium: "Stalden")
09:00-09:30 **Sebastian Ørsted** *TBA*
09:45-10:15 **Dasha Poliakova** *Infinity-local systems*
10:30-11:00 **Simone Siclari** *Calabi-Yau equation on the Kodaira-Thurston manifold*
11:15-11:45 **Kelli Francis-Staite** (Oxford) *What is a C^infinity Scheme (with corners)?*
12:00-13:00 Lunch
13:00-13:30 **Aaron Brookner** *The 10-fold way*
13:45-14:15 **William Petersen** *Quantum Invariants*
14:30-15:00 **Giovanni Russo** *Graphs and geometry of 6-dimensional nearly Kähler manifolds with 2-torus action*
15:15-15:45 **Calvin McPhail-Snyder** *Diagrammatic algebras and categories in quantum algebra*
16:00-16:30 **Alessandro Malusà** *Geometric quantisation and Andersen-Kashaev theory*
'SENIOR/POSTDOC DISCUSSION DAY' Lecture room: "Stalden" - 1 floor
09:30-12:00 Senior/postdoc discussions
12:00-13:00 Lunch
13:15-14:15 Senior/postdoc discussions
14:30-15:30 Senior/postdoc discussions

18:30 Special dinner *(three courses and wine)*
20:00 Cheese & wine in the library

AARHUS UNIVERSITY
Danmarks Grundforskningsfond Danish National Research Foundation

Monday 20 November (Auditorium: "Stalden")

09:30-10:30 **Yang Huang** *Rigidity vs Flexibility in Contact topology*

10:45-11:45 **Gabriele Rembado** *Hitchin connection for complex Chern-Simons theory in genus one*

11:45-13:00 Photo session and lunch

13:15-14:15 **Omar Kidwai** *Higher length-twist coordinates and applications - effective superpotentials from the geometry of opers*

14:30-15:30 **Gus Schrader** *Continuous tensor categories from quantum higher Teichmüller theory*

15:45-16:45 **Sergey Arkhipov** *Braid relations in the affine Hecke category and differential forms with logarithmic singularities*

17:00-18:00 **Richard Wentworth** *Asymptotics in the Hitchin moduli space*

18:00 Dinner

19:15 **Jane Jamshidi** *Publications and funding (BFI points)*

20:00 Cheese & wine in the Library

Tuesday 21 November (Auditorium: "Stalden")

09:30-10:30 **Qiongling Li** *On cyclic Higgs bundles*

10:45-11:45 **Roberta Iseppi** *BRST cohomology and ghost fields: a possible geometric interpretation*

12:00-13:00 Lunch

13:15-14:15 **Mykola Dudushenko** *One-dimensional topological sectors in 3d N=4 theories*

14:30-15:30 **Du Pei** *On the mirror symmetry of (B,A,A)-branes*

16:00 Departure

Book of Abstracts

Sergey Arkhipov *Braid relations in the affine Hecke category and differential forms with logarithmic singularities*
We recall the even and odd algebro-geometric realizations of the affine Hecke category - one via equivariant coherent sheaves on the Steinberg variety and the other in terms of some equivariant DG-modules over the DG-algebra of differential forms on a reductive group G. The latter one has a toy analog called the coherent Hecke category. It contains certain canonical objects satisfying braid relations via convolution. The proof uses simple facts from the geometry of Bott-Samelson varieties. Our goal is to provide a similar proof of braid relations in the affine Hecke category. It turns out that canonical braid group generators are given by certain DG-modules of logarithmic differential forms and braid relations follow immediately from a general statement which seems to be new: direct image of the DG-module of logarithmic differential forms does not depend on a resolution of singularities.

Gergely Bérczi *Non-reductive moduli spaces and their cohomology*
This is a short report on recent progress in construction of GIT quotients of projective varieties by non-reductive algebraic group actions and description of their cohomology and intersection numbers using symplectic geometry. We review some examples and applications.

Gaëtan Borot *Geometric recursion*
I will present a new formalism, which takes as input a functor E from a category of surfaces with their mapping classes as morphisms, to a category of topological vector spaces, together with glueing operations, as well as a small amount of initial data, and produces as output functorial assignments S -> \Omega_S in E(S). This construction is done by summing over all excisions of homotopy class of pair of pants decompositions of S, and we call it ''geometric recursion''. The topological recursion of Eynard and Orantin appears as a projection of the geometric recursion when E(S) is chosen to be the space of continuous functions over the Teichmuller space of S, valued in a Frobenius algebra -- and the projection goes via integration over the moduli space. More generally, the geometric recursion aims at producing all kinds of mapping class group invariant quantities attached to surfaces. This is joint work with Jorgen Ellegaard Andersen and Nicolas Orantin.

Aaron Brookner *The 10-fold way*
Various classification problems have 10 possibilities as their answer, and the fact that they're related is what is known as "the 10-fold way". I'll summarize how Altland & Zirnbauer's classification of the phases of free, gapped fermions according to their CT symmetries has 10 answers; and how this relates to (Morita classes of) the Clifford algebras over R & C, and the 10 division super-algebras.

Mykola Dudushenko *One-dimensional topological sectors in 3d N=4 theories*
I will describe techniques for exact computations of a large class of correlation functions in 3d N=4 gauge theories placed on S^3. In general, N=4 theories on S^3 have interesting supersymmetric sectors, defined by passing to the cohomology of a certain supercharge, and described by an emergent 1d theory. There are

two such 1d sectors that are exchanged by the mirror symmetry: one related to the Higgs branch and another related to the Coulomb branch. When the corresponding branch is not lifted, the 1d theory is topological. This is a 3d analog of the chiral algebra in 4d N=2 SCFTs discovered by Beem et. al. Moreover, our construction provides a generalization away from the conformal point in 3d case. (Which is still missing in the 4d case!) . We use supersymmetric localization to access the 1d sectors. The final result can be encoded as a star-product on the corresponding branch. One of the many applications is that the Coulomb branch result (so far, only for abelian theories) provides a proof of the known descriptions of Coulomb branches.

Kelli Francis-Staite *What is a C^infinity Scheme (with corners)?*
C^infinity schemes are a generalisation of the category of manifolds using algebraic geometry language. They contain all fibre products of manifolds, which the ordinary category of manifolds does not. In this talk, I will describe this category and how it fits into the picture of derived geometry. If time, I will also explain how to extend these ideas to manifolds with corners.

Yang Huang *Rigidity vs Flexibility in Contact topology*
In this talk we will first survey both the rigid (holomorphic curves) and flexible (h-principle) phenomena in contact topology, and then speculate on the borderline between the two. Joint work with K. Honda. Omar Kidwai Higher length-twist coordinates and applications - effective superpotentials from the geometry of opers. We describe joint work with L. Hollands on the geometry of the moduli space of flat connections over a Riemann surface. On the one hand, we generalize and compute certain "complexified Fenchel-Nielsen" coordinates for SL(2)-connections to higher rank using the spectral network "abelianization" approach of Gaiotto-Moore-Neitzke. We then use these coordinates to compute superpotentials, following a conjecture of Nekrasov-Rosly-Shatashvili which roughly states the following: a certain low energy effective twisted superpotential arising from compactifying a theory of class S is equal to the generating function (in the sense of symplectic geometry), in some special coordinates, of the Lagrangian submanifold of opers in the associated moduli space of flat connections.

Roberta Iseppi *BRST cohomology and ghost fields: a possible geometric interpretation*
The BRST cohomology complex plays a very important role in facing the problem of quantizing non-abelian gauge theories via the path integral approach. Indeed, this quantization procedure fails when applied to gauge theories, where the presence of local symmetries causes the degeneracy of the propagator. This problem is overcame by introducing extra (non-physical) fields, called ghost-fields, and defining the so-called BRST cohomology complex. It is precisely this cohomology complex that allows the recovery of important information on the theory, such as its set of observables or its renormalizability.

In this talk we will explain how to construct the BRST complex for a U(2)-gauge theory.
Then, we will introduce a notion of generalized Lie algebra cohomology, which allows to give a different description of the BRST complex and to detect a double complex structure. Finally, we will present a possible geometric interpretation of the BRST complex and of the ghost fields, which would relate them with a resolution of singularities procedure.

Omar Kidwai *Higher length-twist coordinates and applications - effective superpotentials from the geometry of opers*

We describe joint work with L. Hollands on the geometry of the moduli space of flat connections over a Riemann surface. On the one hand, we generalize and compute certain "complexified Fenchel-Nielsen" coordinates for SL(2)-connections to higher rank using the spectral network "abelianization" approach of Gaiotto-Moore-Neitzke. We then use these coordinates to compute superpotentials, following a conjecture of Nekrasov-Rosly-Shatashvili which roughly states the following: a certain low energy effective twisted superpotential arising from compactifying a theory of class S is equal to the generating function (in the sense of symplectic geometry), in some special coordinates, of the Lagrangian submanifold of opers in the associated moduli space of flat connections.

Tobias Kildetoft *An introduction to 2-categories and categorification*
I will give a gentle introduction to finitary 2-categories and categorification. The focus will all the way through be on 2-categories with one object to ease the notation, and the main example throughout will be Soergel bimodules. At the end, I will discuss some recent applications of the theory to the representation theory of semisimple Lie algebras as well as the status of some open problems.

Qiongling Li *On cyclic Higgs bundles*
Given a polystable G-Higgs bundle over a Riemann surface, there is an equivariant harmonic map f into the symmetric space G/K through solving the Hitchin equation. Under the suitable condition, f is minimal. We derive a maximum principle for a type of the elliptic systems and apply it to analyze the Hitchin equation for the cyclic Higgs bundles. We show several monotonicity and domination results about the pullback metric of the (branched) minimal immersion f. Also, we obtain a lower and upper bound of the extrinsic curvature of the image of the map f. This is joint work with Song Dai.

Alessandro Malusà *Geometric quantisation and Andersen-Kashaev theory*
The SL(2,C)-Chern-Simons theory can be approached in different ways, two of which are geometric quantisation on moduli spaces and Andersen-Kashaev Teichmüller TQFT. Unlike in the case of SU(2), the relation between these different formulations is not yet clear. In the case of a genus one surface, however, the quantum Hilbert spaces coming from these two viewpoints are explicitly isomorphic via the Weil-Gel'fand-Zak transform. In a soon-to-appear work with Jørgen, we use geometric quantisation to define quantum operators associated to certain elements of the algebra of regular functions on the moduli space, and use the above-mentioned transform to obtain operators in Teichmüller theory. We then proceed to show an analogous of the AJ-conjecture for these operators and the invariants produced by the TQFT for the first two hyperbolic knots.

Paolo Masulli *The topology of neuronal networks and their activation patterns*
The dynamical evolution of a network is strongly associated with its pattern of internal connections, but the lack of periodic patterns in the vast majority of biological networks, and in recurrent neural networks in particular, makes it difficult to understand this correlation from a theoretical and formal approach.
We use algebraic topology to encode the connectivity structure of a network and build invariants giving information on its dynamical evolution, in order to relate topology and activation patterns. Our final goal is to shed light on the problem of how the more complex temporal activation patterns that are observed in biological networks are related with their topology.

Du Pei *On the mirror symmetry of (B,A,A)-branes*
Picking a real form of a complex Lie group G defines a "(B,A,A)-brane" inside the moduli space of *G*-Higgs bundles. In this talk, I will discuss construction of mirrors of such branes, and show how these proposals can be tested.

William Petersen *Quantum Invariants*
Quantum Invariants are invariants of closed oriented three-manifolds that extends to a topological quantum field theory (TQFT). Originally, interest in TQFTs arose through Edward Witten's study of Chern Simons theory. In this talk, I shall present the axioms of a TQFT, touch upon how Witten's study leads to a conjecture concerning quantum invariants known as the asymptotic expansion conjecture (AEC), and if time permits it, I shall report on recent progress with regard to the AEC joint with my supervisor J.E. Andersen.

Dasha Poliakova *Infinity-local systems*

I will introduce and discuss the notion of an infinity-local system on a simplicial set taking values in an arbitrary DG category. For a topological space X, the DG category of infinity-local systems of chain complexes on its singular set is quasiequivalent to the DG category of modules over the algebra of chains on the loop space of X, as is shown in my bachelor thesis.

Calvin McPhail-Snyder *Diagrammatic algebras and categories in quantum algebra*
Diagram algebras" presented as linear combinations of of diagrams subject to local relations arise throughout representation theory and topology. They provide interesting elementary examples of linear monoidal categories, but they are also a powerful tool for constructing invariants of topological objects like knots, graphs, and links. In my talk I will give examples of these categories and discuss some applications.

Gabriele Rembado *Hitchin connection for complex Chern-Simons theory in genus one*
We construct a Hitchin connection for the Kaehler quantisation of the moduli space of -Higgs bundle over elliptic curves, at any quantum level. We discuss its compatibility with Hitchin's circle action on the moduli space, and we compare it with the Hitchin-Witten connection using the Bargmann transform.

Giovanni Russo *Graphs and geometry of 6-dimensional nearly Kähler manifolds with 2-torus action*
In this talk we firstly introduce the notions of nearly Kähler manifold and G2-structure on the relative cone, presenting some known examples. We consider then a T^2-action on each of them and construct particular graphs, depending on the dimension of the stabilizers of the action. It turns out that these graphs are trivalent, and this can be proved abstractly when the action is effective. Finally, we construct explicitly the moment map given by the (nearly) Kähler form and the generating vector fields of the T^2-action and study the geometry on the quotients by T^2 of its level sets.

Cristiano Spotti/Martin Borbon *Singular KE metrics, "stable" tangent cones of algebraic varieties and applications*
After recalling results on the existence of Kähler-Einsten (KE) metrics on smooth manifolds, we will discuss extension to singular varieties and pairs of varieties with R-divisors. In particular, we will focus on questions related to the metric behaviour near the singular locus based on recent developments relating metric tangent cones to valuations and "stability" of klt singularities. We will present the following applications of the previous ideas:
- Existence of KE metrics on explicit families of Fano varieties (this is the first talk, based on work of Spotti-Sun).
- Calabi-Yau metrics with conical singularities along line arrangements in the projective plane and a conjectural Chern-Weil formula (this is the second talk, based on on-going work of de Borbon-Spotti).

Gus Schrader *Continuous tensor categories from quantum higher Teichmüller theory*
I'll describe recent joint work with Alexander Shapiro in which we apply tools from quantum higher Teichmuller theory to prove the conjecture of Frenkel and Ip that the category of principal series representations of the modular double of U_q(sl(n)) is closed under tensor products in the continuous sense. Our result generalizes the work of Ponsot and Teschner on the case of the rank 1 quantum group U_q(sl(2)).