AARHUS UNIVERSITY
DEPARTMENT OF MATHEMATICS
CENTRE FOR QUANTUM GEOMETRY O FMODULI SPACES

Conference: New Developments in TQFT

Venue: Auditorium D1 (1531-113)

Programme

Monday 27 July

09.00-09.30: Coffee/tea by Aud. D1

09.30-10.30: **Gregor Masbaum:** An application of TQFT to modular representation theory

11.00-12.00: **Thang Le:** On the unified quantum invariant of integral homology 3-spheres associated to simple Lie algebras

12.00-13.30: Lunch break

13.30-14.30: **Nathan Dunfield:** A tale of two norms

14.45-15.45: **Satoshi Nawata:** Knot invariants, BPS states and A-polynomials

16:00-17:00: **Jürgen Fuchs:** Defects in Reshetikhin-Turaev TFTs and an obstruction in the Witt group

18.00: Networking dinner in the Math Canteen

Tuesday 28 July

09.00-09.30: Coffee/tea by Aud. D1

09.30-10.30: **Rinat Kashaev:** Quantum dilogarithm and self-dual LCA groups

11.00-12.00: **Francesco Costantino:** Non semi simple TQFTs Part 1

12.00-13.30: Lunch break

13.30-14.30: **Christopher Schommer-Pries:** Extended topological field theory in dimension three

14.45-15.45: **Tian Yang:** Volume conjectures for Reshetikhin-Turaev and Turaev-Viro invariants

16.00-17.00: **Christian Zickert:** Coordinates for representations of 3-manifold groups

Wednesday 29 July

09.00-09.30: Coffee/tea by Aud. D1

09.30-10.30: **Sergei Gukov:** TQFT in the 21st century

11.00-12.00: **Tudor Dimofte:** Towards categorification in complex Chern-Simons theory

12.00-13.30: Lunch break

13.30-14.30: **Ruth Lawrence:** Explicit DGLA models of simple chain complexes and their properties

14.45-15.45: **Roland van der Veen:** Quantum coinvariants

16:00-17:00: **Christoph Schweigert:** Surface defects in Turaev-Viro theories, Brauer-Picard groups and some relations to representation theory

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Venue: Auditorium D1 (1531-113)

Thursday 30 July

09.00-09.30: Coffee/tea by Aud. D1

09.30-10.30: **Stavros Garoufalidis:** Evaluation of state integrals via Grothendieck residues

11.00-12.00: **Kazuo Habiro:** Functors on Lagrangian cobordisms associated to ribbon Hopf algebras

12.00-13.30: Lunch break

13.30-14.30: **Gaetan Borot:** Modular functors, cohomological field theories and topological recursion

14.45-15.45: **Bruno Martelli:** An analytic family of representations for the mapping class group of punctured surfaces

16:00-17:00: **Steven Bradlow:** Higgs bundle moduli spaces in TQFT

Friday 31 July

09.00-09.30: Coffee/tea by Aud. D1

09.30-10.30: **Dror Bar-Natan:** Polynomial Time Knot Polynomials
(Handouts available at: http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1507/)

11.00-12.00: **Jun Murakami:** Logarithmic invariants of knots in a three manifold

12.00-13.30: Lunch break

13.30-14.30: **Christian Blanchet**: Non semi simple TQFTs Part 2

ABSTRACTS

Dror Bar-Natan, University of Toronto

Title: Polynomial Time Knot Polynomials

Abstract: The value of things is inversely correlated with their computational complexity. "Real time" machines, such as our brains, only run linear time algorithms, and there's still a lot we don't know. Anything we learn about things doable in linear time is truly valuable. Polynomial time we can in-practice run, even if we have to wait; these things are still valuable. Exponential time we can play with, but just a little, and exponential things must be beautiful or philosophically compelling to deserve attention. Values further diminish and the aesthetic-or-philosophical bar further rises as we go further slower, or un-computable, or ZFC-style intrinsically infinite, or large-cardinalish, or beyond.
I will explain some things I know about polynomial time knot polynomials and explain where there's more, within reach.

Christian Blanchet, University Paris Diderot

Title: Non semi simple TQFTs Part 2

Abstract: In this second part on non semi-simple TQFTs, we will give details about the universal construction and show that Costantino-Geer-Patureau invariants have graded TQFT extensions. We will describe the TQFT vector spaces and discuss the underlying representations of mapping class groups. Joint work with Francois Costantino, Nathan Geer and Bertrand Patureau.

Gaetan Borot, Max Planck Institute for Mathematics, Bonn

Title: Modular functors, cohomological field theories and topological recursion.

Abstract: Topological modular functors were introduced more than 25 years ago by Segal as a axiomatization of rational conformal field theory. Given any unitary topological modular functor, I will explain how to construct a vector bundle over the moduli space of curves, whose Chern class defines a cohomological field theory. The intersection of this Chern class with psi-classes in M_{g,n} can be computed by the topological recursion of Eynard and Orantin. The initial data of the recursion involve the Dehn twists, the central charge and the S-matrix prescribed by the modular functor. The cohomological field theory associated to the Verlinde bundle of WZW conformal blocks -- already studied by Marian, Oprea, Pandharipande, Pixton and Zvonkine -- is a special case. This is based on a joint work with Jorgen Ellegaard Andersen and Nicolas Orantin.

Steven Bradlow, UIUC

Title: Higgs bundle moduli spaces in TQFT

Abstract: Higgs bundles and their moduli spaces have many remarkable properties, by virtue of which they impact surprisingly diverse areas of mathematics including TQFT. We will review some of the basic constructions and describe their role in applications.

Francesco Costantino, Université de Toulouse

Title: Non semi-simple TQFTs Part 1

Abstract: This is the first talk of a series of two on the construction of the non semi-simple TQFTs, and it will precede C. Blanchet's talk.
We will discuss the structure of the category of representations of the version of U_qsl_2 used in our construction of invariants of 3 manifolds equipped with cohomology classes, build jointly with Nathan Geer and Bertrand Patureau.
Then we will explain what are the main differences with respect to the standard Reshetikhin-Turaev construction and how to overcome it to get these invariants.

Tudor Dimofte, Institute for Advanced Study

Title: Towards categorification in complex Chern-Simons theory

Abstract: I will present one perspective on what categorification of complex Chern-Simons theory should mean and the structure it should have, inspired by three- and four-dimensional supersymmetric gauge theory. Some elementary examples include categorification of the hyperbolic structure on a tetrahedron, and a categorification of the 2-3 move.

Nathan Dunfield, University of Illinois

Title: A tale of two norms.

Abstract: The first cohomology of a hyperbolic 3-manifold has two natural norms: the Thurston norm, which measure topological complexity of surfaces representing the dual homology class, and the harmonic norm, which is just the L^2 norm on the corresponding space of harmonic 1-forms. Bergeron-Sengun-Venkatesh recently showed that these two norms are closely related, at least when the injectivity radius is bounded below. Their work was motivated by the connection of the harmonic norm to the Ray-Singer analytic torsion and issues of torsion growth in homology of towers of finite covers. After carefully introducing both norms, I will discuss new results that refine and clarify the precise relationship between them; one tool here will be a third norm based on least-area surfaces. This is joint work with Jeff Brock.

Jürgen Fuchs, Karlstad University

Title: Defects in Reshetikhin-Turaev TFTs and an obstruction in the Witt group

Abstract: We extend three-dimensional topological field theories of Reshetikhin-Turaev type based on arbitrary modular tensor categories to three-manifolds with boundaries and embedded surfaces, also called surface defects. The surface defects between two three-dimensional regions labeled by specified modular tensor categories are classified in terms of the bicategory of module categories over a suitable fusion category. The existence of that fusion category is obstructed; the obstruction takes values in the Witt group of modular tensor categories. As an illustration of possible applications to topological phases in condensed matter physics, we determine ground state degeneracies and the spectrum of generalized quasi-particle excitations for multi-layer topological phases in the presence of permutation twist defects. (Joint with C. Schweigert and A. Valentino).

Stavros Garoufalidis, Georgia Institute of Technology

Title: Evaluation of state integrals via Grothendieck residues

Abstract: State integrals appear in abundance in quantum topology in the work of Hikami, Kashaev, Andersen-Kashaev, Luo-Kashaev-Vartanov and others. They are analytic functions on a cut complex plane, regular at 1. We prove that the evaluation of state integrals at 1 are given by semiclassical invariants. Our proof uses multivariate complex analysis and Grothendieck residues. Joint work with Rinat Kashaev, done during a visit to Aarhus University in July 2015.

Sergei Gukov, Caltech

Title: TQFT in the 21st century

Abstract: Einstein used to say "If you can't explain it simply, you don't understand it well enough." Keeping this in mind, I will attempt to survey physics approaches to categorification (refinement) of 2d and 3d TQFTs developed in 80's and 90's, illustrating how each one helps to do calculations that could not be done previously.

Kazuo Habiro, Kyoto University

Title: Functors on Lagrangian cobordisms associated to ribbon Hopf algebras

Abstract: Crane-Yetter and Kerler, independently, defined the braided monoidal category Cob of 3-dimensional cobordisms between connected surfaces with boundaries parametrized by S^1. The Hennings TQFT, defined by Kerler, is an extension of the Hennings invariant of closed 3-manifolds to a functor defined on Cob, which is constructed without using representation theory of the quantum group. The Hennings TQFT is defined for each factorizable finite-dimensional ribbon Hopf algebra. The category B of bottom tangles in handlebodies may be regarded as a braided monoidal subcategory of Cob, and contains the Hopf algebra object in Cob. For any ribbon Hopf algebra A, one can define a functor J from B to the category of A-modules. The category L of Lagrangian cobordisms is a subcategory of Cob whose morphisms are Lagrangian cobordisms, or morphisms in Cob which are homologically equivalent to morphisms in B. We consider an extension of the functor J to L. This is done for a ribbon Hopf algebra with some additional data. In particular, we can obtain a functor on L extending the unified quantum invariants of integral homology spheres defined over the cyclotomic completion of the polynomial ring Z[q]. Part of this talk is based on work in progress joint with Thang Le.

Rinat Kashaev, University of Geneva

Title: Quantum dilogarithm and self-dual LCA groups

Abstract: I will review the definition of the quantum dilogarithm over a self-dual (in the sense of Pontryagin) Locally Compact Abelian group and a construction of 3d TQFT with corners on shaped triangulations. The talk is based on joint works with Jørgen Ellegaard Andersen

Ruth Lawrence, Hebrew University of Jerusalem

Title: Explicit DGLA models of simple chain complexes and their properties

Abstract: About twenty years ago, Kontsevich started building an interesting differential on a free Liealgebra with two odd generators a, b and one even generator e. The differential began $da = -\frac{1}{2}[a,a]$, $db = -\frac{1}{2}[b,b]$, $de = b - a + \frac{1}{2}[e, a+b] + \cdots$ with terms involving Bernoulli numbers. It seemed to be a miracle that the higher order terms could be chosen so that $d^2 = 0$. About fifteen years ago, Sullivan realised that such differentials should exist for abstract reasons related to algebraic topology, an argument which works for any cell complex but does not produce a unique or closed formula in general. For the interval, the formula is unique and in joint work with Sullivan we showed how it can be interpreted using the formalism of connections, curvature and gauge transformation from differential geometry.
In this talk we will discuss chain complexes of dimension up to two, and consider the interplay between the algebra and geometry inherent in these constructions including localisation, functorial properties under subdivision (via the Baker-Campbell-Hausdorf formula) and reconstruction of the set of points by solving the Maurer-Cartan equations. This is a report on work in progress joint with Dennis Sullivan and my students Nir Gadish and Itay Griniasty.

Thang Le, Georgia Institute of Technology

Title: On the unified quantum invariant of integral homology 3-spheres associated to simple Lie algebras.

Abstract: We describe the construction of the unified invariant of integral homology 3-spheres with values in the Habiro ring. The invariant unifies all the quantum (Witten-Reshetikhin-Turaev) invariants associated with the simple Lie algebra and represents the quantum invariants as a kind of "analytic function" defined on the set of roots of unity. This is joint work with K. Habiro.

Bruno Martelli, University of Pisa

Title: An analytic family of representations for the mapping class group of punctured surfaces

Abstract: We introduce a family of representations for the mapping class group of a punctured surface, which act on a fixed infinite-dimensional separable Hilbert space and depend analytically on a complex number A with |A| <= 1. The family includes the familiar TQFT finite-dimensional representations (when A is a root of unity distinct from 1,-1,i,-i), the SU(2)-character variety representation (at A=-1), and the multicurve representation (at A=0). We discuss various properties of this family and focus on some open questions. (joint with F. Costantino).

Gregor Masbaum, Paris VI

Title: An application of TQFT to modular representation theory

Abstract: We use TQFT to obtain previously unknown dimension and character formulas for some highest weight modules for symplectic groups over finite fields in the natural characteristic.

Jun Murakami, Waseda University

Title: Logarithmic invariants of knots in a three manifold

Abstract: We first introduce the logarithmic invariants of knots in S^3 from the colored Jones invariant and the colored Alexander invariant and explain a relation to the radical of the restricted quantum group. These invariants can be considered as generalizations of Kasha's invariant, and they are related to the hyperbolic volumes of cone manifolds along the knot. Then we extend logarithmic invariants to invariants of knots in a three manifold by combining the Hennings invariant for a three manifold. The Hennings invariant is constructed by using the right integral of a finite-dimensional Hopf algebra. We also explain their relation to the hyperbolic volume of the manifold.

Satoshi Nawata, Max Planck Institute for Mathematics, Bonn

Title: Knot invariants, BPS states and A-polynomials

Abstract: I will talk about the deep relationships of knot invariants, BPS states and A-polynomials. After the work of Khovanov, there have been a lot of progress on the categorifications of quantum knot invariants. The knot homology can be identified with the space of M5-M2 BPS states that are endowed with rich structure. This in turn enables computation of knot homology for various classes of knots and links and consequent computation of super-A-polynomial - the deformation of the classical A-polynomial. The deformation of A-polynomial encodes luxurious information about mirror geometry, BPS numbers, quantization of symplectic varieties.

Christopher Schommer-Pries, Max Planck Institute for Mathematics, Bonn

Title: Extended topological field theory in dimension three

Abstract: I will survey recent advances in the classification of extended topological field theories, with a particular focus on dimension three. I will describe recent work (joint with B. Bartlett, C. Douglas, and J. Vicary) which gives an explicit "generators and relations" classification of partially extended 3D TFTS (assigning values only to 3-manifolds, surfaces, and 1-manifolds). This will be compared to the fully-local case (which has been considered in joint work with C. Douglas and N. Snyder).

Christoph Schweigert, University of Hamburg

Title: Surface defects in Turaev-Viro theories, Brauer-Picard groups and some relations to representation theory

Abstract: We explain how surface defects in three-dimensional topological field theories of Turaev-Viro type are related to bimodule categories over fusion categories. Dijkgraaf-Witten theories are TFTs of Turaev-Viro type that admit a gauge-theoretic realization. We present a geometric construction of surface defects in these theories; our results perfectly match the general analysis presented in Jürgen Fuchs' contribution, combined with results by Ostrik on bimodule categories. We then explain how symmetries of Turaev-Viro are described in terms of the Brauer-Picard group of the underlying fusion category. This allows us to understand symmetries of Dijkgraaf-Witten theories rather explicitly. We finally give an interpretation of the isomorphism of the Brauer-Picard group of a fusion category and the group of braided autoequivalences of its Drinfeld center in terms of TFTs with defects.

Roland van der Veen, Korteweg-de Vries Institute for Matematics

Title: Quantum coinvariants

Abstract: Instead of studying TQFT and quantum knot invariants coming from quantum groups we propose to shift the viewpoint to the dual quantum group. This emphasizes the non-commutative geometry and invariant theory, shedding some new light on familiar objects such as the Ising model and the colored Jones polynomial

Tian Yang, Stanford University

Title: Volume conjectures for Reshetikhin-Turaev and Turaev-Viro invariants

Abstract: In a joint work with Qingtao Chen, we consider a family of Turaev-Viro type invariants for a 3-manifold M with non-empty boundary, indexed by an integer $r \geq 3$; and propose a volume conjecture for hyperbolic M that these invariants grow exponentially at large r with a growth rate the hyperbolic volume of M: The crucial step is the evaluation at the root of unity $\exp(2\pi\sqrt{(-1/r)})$ instead of that at the usually considered root $\exp(\pi\sqrt{(-1/r)})$. Evaluating at the same root $\exp(2\pi\sqrt{(-1/r)})$, we then conjecture that, the original Turaev-Viro invariants and the Reshetikhin-Turaev invariants of a closed hyperbolic 3-manifold M grow exponentially with growth rates respectively the hyperbolic and the complex volume of M: This uncovers a different asymptotic behavior of the values at other roots of unity than that at $\exp(\pi\sqrt{(-1/r)})$ predicted by Witten's Asymptotic Expansion Conjecture, which may indicate some different geometric interpretation of the Reshetikhin-Turaev invariants than the SU (2) Chern-Simons theory. Numerical evidences will be provided to support these conjectures.

Christian Zickert, University of Maryland

Title: Coordinates for representations of 3-manifold groups

Abstract: We study the shape and Ptolemy varieties of a compact 3-manifold M with a topological ideal triangulation. The varieties give coordinates for representations of pi_1(M) in the sense that each point determines a representation (up to conjugation). We describe the varieties, how to compute them, and how to compute invariants such as trace fields and complex volume.